

Topic :- Electric charges and fields

1

(a)

Here, $q = \pm 6.0 \text{ nC} = \pm 6.0 \times 10^{-9} \text{ C}$

$2a = 6 \text{ cm} = 6 \times 10^{-2} \text{ m}$

$r = 4 \text{ cm (on equatorial line)}$

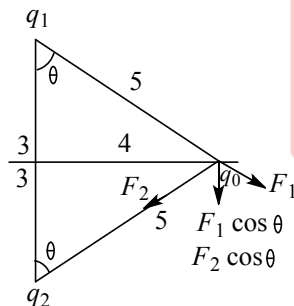
$= 4 \times 10^{-2} \text{ m}$

$q_0 = 2 \text{ nC}$

$= 2 \times 10^{-9} \text{ C}, F = ?$

$F = F_1 \cos \theta + F_2 \cos \theta$

$$= 2 \times \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^2} \cos \theta$$



$$= 2 \times 9 \times 10^9 \times \frac{6 \times 10^{-9} \times 2 \times 10^{-9}}{(5 \times 10^{-2})^2} \times \frac{3}{5}$$

$$= 5.18 \times 10^{-5} \text{ N}$$

$$\vec{F} = -51.8 \hat{j} \mu\text{N}$$

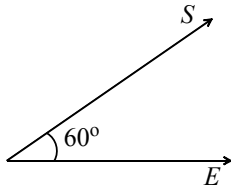
2

(a)

The electric intensity outside a charged sphere.

$$E = \frac{\sigma R^2}{\epsilon_0 r^2}$$

3

(d)Force acting on the charged particle = $q\vec{E}$ Work done in moving a distance S ,

$$W = q\vec{E} \cdot \vec{S} = (qE) \times S \times \cos \theta$$

$$10J = (0.5 C) \times E \times 2 \cos 60^\circ$$

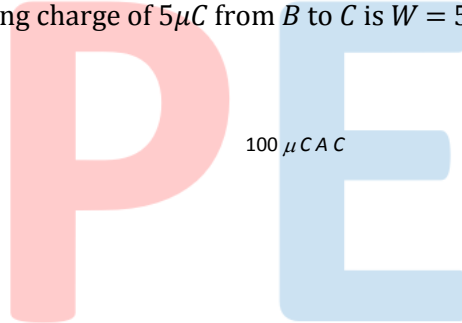
$$\Rightarrow E = 10 \times 2 = 20 \text{ NC}^{-1} = 20 \text{ Vm}^{-1}$$

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(d)

$$\frac{\sigma_{small}}{\sigma_{Big}} = \frac{q}{Q} \times \frac{R^2}{r^2} = \frac{q}{(nq)} \times \frac{(n^{1/3}r)^2}{r^2} = n^{-1/3} = (64)^{-1/3} = \frac{1}{4}$$

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(d)Work done in displacing charge of $5\mu\text{C}$ from B to C is $W = 5 \times 10^{-6}(V_C - V_B)$ where

$$V_B = 9 \times 10^9 \times \frac{100 \times 10^{-6}}{0.4} = \frac{9}{4} \times 10^6 \text{V}$$

$$\text{and } V_C = 9 \times 10^9 \times \frac{100 \times 10^{-6}}{0.5} = \frac{9}{5} \times 10^6 \text{V}$$

$$\text{so } W = 5 \times 10^{-6} \times \left(\frac{9}{5} \times 10^6 - \frac{9}{4} \times 10^6 \right) = -\frac{9}{4} \text{J}$$

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(c)Here, $\theta = 60^\circ$, $E = 10^5 \text{NC}^{-1}$

$$\tau = 8\sqrt{3} \text{ Nm}, q = ?, 2a = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$$

$$\text{From } \tau = pE \sin \theta = q(2a)E \sin \theta$$

$$q = \frac{\tau}{2aE \sin \theta} = \frac{8\sqrt{3}}{2 \times 10^{-2} \times 10^5 \times \sin 60^\circ}$$

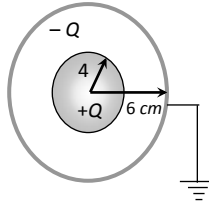
$$= \frac{8\sqrt{3}}{2 \times 10^3 \times \sqrt{3}/2}$$

$$q = 8 \times 10^{-3} \text{ C}$$

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(d)

Suppose charge on inner sphere is $+Q$ as shown. Potential on inner sphere



$$V = \frac{Q}{4} - \frac{Q}{6}$$

$$\Rightarrow 3 = Q\left(\frac{1}{4} - \frac{1}{6}\right) \Rightarrow Q = 36 \text{ e.s.u}$$

8

(c)

Solid angle, $\Omega = \frac{A}{r^2}$

$$= \frac{\pi R^2}{r^2} \Rightarrow \frac{\pi \times (1)^2}{10^6} \Rightarrow 0.00018^\circ$$

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(b)

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} = \frac{q_1(q_1 - q_2)}{4\pi\epsilon_0 r^2}$$

$\therefore F$ will be maximum, if

$$\frac{dF}{dq_1} = 0$$

$$\therefore q - 2q_1 = 0 \text{ or } q_1 = q/2 \text{ or } q_1/q = 0.5$$

10

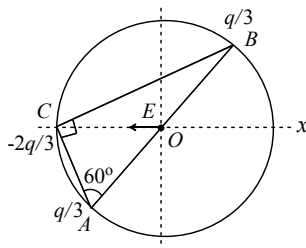
(a)

$$E = \frac{V}{d} = \frac{30 - (-10)}{(2 \times 10^{-2})} = 2000 \text{ V/m}$$

11

(c)

Net electric field due to both charges $q/3$, will get cancelled. Electric field due to $\left(\frac{-2q}{3}\right)$ will be directed in $-ve$ axis



$$E = \frac{k\left(\frac{2q}{3}\right)}{R^2} \Rightarrow E = \frac{q}{6\pi\epsilon_0 R^2}$$

$$\text{P.E. of system} = \frac{K\left(\frac{q}{3}\right)^2}{2R} + \frac{K\frac{q}{3}\left(-\frac{2q}{3}\right)}{2R \sin 60^\circ} + \frac{K\frac{q}{3}\left(-\frac{2q}{3}\right)}{2R \cos 60^\circ}$$

P.E. of system $\neq 0$

Force between B and C

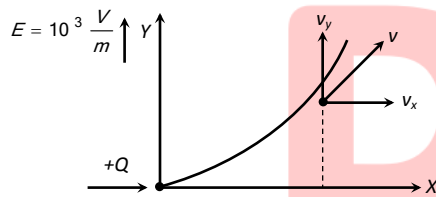
$$F = \frac{K\left(\frac{2q}{3}\right)\left(\frac{q}{3}\right)}{\left(2R \sin 60^\circ\right)^2} = \frac{4 \times 2Kq^2}{9 \times 4 \times 3R^2} = \frac{2q^2}{9 \times 3 \times 4\pi\epsilon_0 R^2}$$

$$(\text{attractive}) = \frac{1}{54} \frac{q^2}{\pi \epsilon_0 R^2}$$

$$\text{Potential at O, } V = \frac{K\left(\frac{q}{3} + \frac{q}{3} - \frac{2q}{3}\right)}{R} = 0$$

12 **(c)**

Body moves along the parabolic path



For vertical motion : By using $v = u + at$

$$\Rightarrow v_y = 0 + \frac{QE}{m} \cdot t = \frac{10^{-6} \times 10^3}{10^{-3}} \times 10 = 10 \text{ m/sec}$$

For horizontal motion - It's horizontal velocity remains the same *i.e.* after 10 sec, horizontal velocity of body $v_x = 10 \text{ m/sec}$

$$\text{Velocity after 10 sec } v = \sqrt{v_x^2 + v_y^2} = 10\sqrt{2} \text{ m/sec}$$

13 **(b)**

$$\text{Capacitance of a cylindrical capacitor} = \frac{2\pi\epsilon_0 L}{\ln(b/a)}$$

Energy stored in the capacitor

$$\frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{Q^2 \ln(b-a)}{2\pi\epsilon_0 L} = [\text{const}] \frac{Q^2}{L}$$

If the charge is doubled and length is doubled,

$$[\text{const}] \frac{Q^2}{L} = \frac{4(Q^2)}{2(L)} = 2 \text{ times the energy}$$

14 **(c)**

Electric field

$$E = -\frac{d\phi}{dt} = -2ar$$

By Gauss's theorem

$$E(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$q = -8\pi\epsilon_0 ar^2$$

$$\begin{aligned}\rho &= \frac{dq}{dV} = \frac{dq}{dr} \times \frac{dr}{dV} \\ &= (-24\pi\epsilon_0 ar^2) \times \frac{1}{4\pi r^2}\end{aligned}$$

$$\rho = -6\epsilon_0 a$$

15 **(b)**
Because metals are good conductor

16 **(a)**

$$\begin{aligned}E &= -\frac{dV}{dx} = -\frac{d}{dx}(5x^2 + 10x - 9) = -10x - 10 \\ \therefore (E)_{x=1} &= -10 \times 1 - 10 = -20V/m\end{aligned}$$

17 **(a)**

$$\begin{aligned}\text{Work done } W &= Q(V_B - V_A) \Rightarrow (V_B - V_A) = \frac{W}{Q} \\ &= \frac{10 \times 10^{-3}}{5 \times 10^{-6}} J/C = 2 kV\end{aligned}$$

18 **(b)**
The given arrangement is equivalent to the parallel combination of three identical capacitors. Hence equivalent capacitance $= 3C = 3\frac{\epsilon_0 A}{d}$

19 **(c)**
Inside a conducting body, potential is same everywhere and equals to the potential of it's surface

20 **(c)**
 $c \propto r \Rightarrow C = 4\pi\epsilon_0 R$

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	A	D	D	D	C	D	C	B	A
Q.	11	12	13	14	15	16	17	18	19	20
A.	C	C	B	C	B	A	A	B	C	C

PE