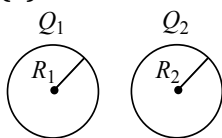


Topic :- Electric charges and fields

1

(a)



Let Q_1 and Q_2 are the charges on sphere of radii R_1 and R_2 respectively. Surface charge density $\sigma = \frac{\text{Charge}}{\text{Area}}$

According to given problem, $\sigma_1 = \sigma_2$

$$\frac{Q_1}{4\pi R_1^2} = \frac{Q_2}{4\pi R_2^2}$$

$$\therefore \frac{Q_1}{Q_2} = \frac{R_1^2}{R_2^2} \quad \dots(i)$$

In case of a charged sphere, $V_s = \frac{1}{4\pi\epsilon_0} \frac{Q}{R}$

$$\therefore V_1 = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{R_1}, V_2 = \frac{1}{4\pi\epsilon_0} \frac{Q_2}{R_2}$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{Q_1}{R_1} \times \frac{R_2}{Q_2} = \frac{Q_1}{Q_2} \times \frac{R_2}{R_1}$$

$$= \left(\frac{R_1}{R_2}\right)^2 \times \left(\frac{R_2}{R_1}\right) = \frac{R_1}{R_2} \quad \dots[\text{Using}(i)]$$

2

(c)

The magnitude of electric field in the annular region of a charged cylindrical capacitor is given by $E = \frac{1}{2\pi\epsilon_0 r} \lambda$ where λ is the charge per unit length and r is the distance from the axis of the cylinder. Thus $E \propto \frac{1}{r}$

3

(c)

Potential energy depends upon the charge at peaks of irregularities. Since every event in universe leads to the minimisation of energy

4

(c)

By using $Q = ne \Rightarrow Q = +2e = +3.2 \times 10^{-19} C$

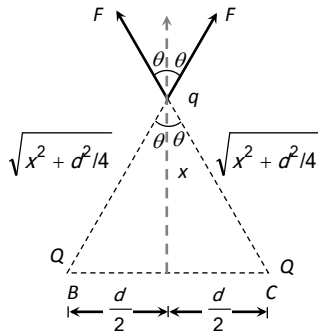
5

(c)

Suppose third charge is similar to Q and it is q

So net force on it

$$F_{net} = 2F \cos \theta$$



$$\text{Where } F = \frac{1}{4\pi\epsilon_0} \cdot \frac{Qq}{\left(x^2 + \frac{d^2}{4}\right)} \text{ and } \cos \theta = \frac{x}{\sqrt{x^2 + \frac{d^2}{4}}}$$

$$\therefore F_{net} = 2 \times \frac{1}{4\pi\epsilon_0} \cdot \frac{Qq}{\left(x^2 + \frac{d^2}{4}\right)} \times \frac{x}{\left(x^2 + \frac{d^2}{4}\right)^{1/2}}$$

$$= \frac{2Qqx}{4\pi\epsilon_0 \left(x^2 + \frac{d^2}{4}\right)^{3/2}}$$

For F_{net} to be maximum $\frac{dF_{net}}{dx} = 0$

$$\text{i.e. } \frac{d}{dx} \left[\frac{2Qqx}{4\pi\epsilon_0 \left(x^2 + \frac{d^2}{4}\right)^{3/2}} \right] = 0$$

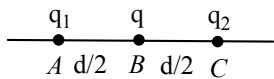
$$\text{or } \left[\left(x^2 + \frac{d^2}{4}\right)^{-3/2} - 3x^2 \left(x^2 + \frac{d^2}{4}\right)^{-5/2} \right] = 0$$

$$\text{i.e. } x = \pm \frac{d}{2\sqrt{2}}$$

6

(d)

For equilibrium, we have



$$F_{AB} + F_{AC} = 0$$

$$\text{or } \frac{1}{4\pi\epsilon_0} \frac{q_1 q}{(d/2)^2} + \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{d^2} = 0$$

Given, $q_1 = q_2 = -1\mu\text{C}$

So, $-\frac{q}{(d/2)^2} + \frac{1}{d^2} = 0$

or $q = \frac{1}{4} = 0.25 \text{ C}$

7

(a)

Initial energy $= \frac{1}{2} \times 1 \times 10^{-6} \times (30)^2 = 450 \times 10^{-6} \text{ J}$

Final energy $= \frac{1}{2}(C_1 + C_2)V_{\text{common}}^2$ [$\because V = \frac{V_1C_1 + V_2C_2}{C_1 + C_2}$]

$= \frac{1}{2} \times 3 \times 10^{-6} \times (10)^2$

$= 150 \times 10^{-6} \text{ J}$

Loss of energy $= (450 - 150) \times 10^{-6} \text{ J} = 300 \times 10^{-6} \text{ J}$

$= 300\mu\text{J}$

8

(a)

From Gauss's law,

$\frac{\text{charge enclosed}}{\epsilon_0}$

= Flux leaving the surface

$\frac{q}{\epsilon_0} = \phi_2 - \phi_1$

$\Rightarrow q = (\phi_2 - \phi_1)\epsilon_0$

PEE

9

(a)

The force acting on the electron $= e.E$

Acceleration of the electron $= \frac{eE}{m}$

$\overline{+ + + + + + +}$

$E = 10^4 \text{ N/C}$
 $\overline{- - - - -}$
 $\uparrow e$

$u = 0, v = ? \quad v^2 - u^2 = 2aS$

$S = 2 \times 10^{-2} \text{ m}$

$\therefore v^2 = 2\left(\frac{e}{m}\right)E \times 2 \times 10^{-2} \text{ m}$

$\frac{e}{m} = 1.76 \times 10^{11} \text{ coulomb/kg}$

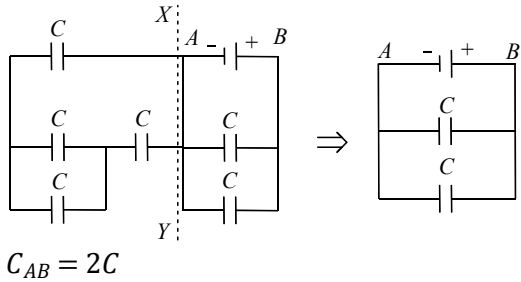
$\therefore v^2 = 2 \times 1.76 \times 10^{11} \times 10^4 \times 2 \times 10^{-2}$

$= 7.04 \times 10^{13} = 70.4 \times 10^{12}$

$\therefore v \approx 0.85 \times 10^7 \text{ m/s}$

10 **(d)**
The work done is given by $= q(V_2 - V_1) = 0$

11 **(d)**
All capacitor lying in left side of line XY are short circuited so circuit can be reduced as follows



12 **(c)**
The energy will be minimum in this case and every system tends to possess minimum energy

13 **(c)**
Work done $= \frac{1}{2} \left(\frac{3C}{2} \right) V^2 = \frac{3CV^2}{4}$

14 **(a)**

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \left(\frac{\epsilon_0 A}{x} \right) V^2$$

$$\therefore \frac{dU}{dx} = \frac{1}{2} \epsilon_0 AV^2 \left(-\frac{1}{x^2} \frac{dx}{dt} \right) \Rightarrow \frac{dU}{dt} \propto x^{-2}$$

15 **(d)**
As $\sigma_1 = \sigma_2$

$$\therefore \frac{Q_1}{4\pi r_1^2} = \frac{Q_2}{4\pi r_2^2}$$
 Or $\frac{Q_1}{4\pi \epsilon_0 r_1^2} = \frac{Q_2}{4\pi \epsilon_0 r_2^2}$

$$\therefore E_1 = E_2 \text{ or } E_1/E_2 = 1$$

16 **(a)**
The total energy before connection

$$= \frac{1}{2} \times 4 \times 10^{-6} \times (50)^2 + \frac{1}{2} \times 2 \times 10^{-6} \times (100)^2$$

$$= 1.5 \times 10^{-2} J$$

When connected in parallel

$$4 \times 50 + 2 \times 100 = 6 \times V \Rightarrow V = \frac{200}{3}$$

Total energy after connection

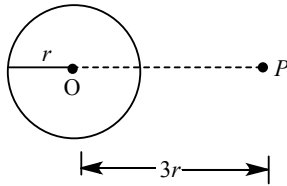
$$= \frac{1}{2} \times 6 \times 10^{-6} \times \left(\frac{200}{3}\right)^2 = 1.33 \times 10^{-2} J$$

18

(d)

The potential at a distance r , due to charge q is

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$$



Potential at a distance ($3r$) is

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{3r}$$

Difference in potential

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{1}{3r} \right]$$

$$\Rightarrow V = \frac{2q}{4\pi\epsilon_0 \times 3r}$$

Intensity of electric field

$$E = \frac{q}{4\pi\epsilon_0} \frac{1}{(3r)^2}$$

$$\therefore \frac{E}{V} = \frac{q}{4\pi\epsilon_0 qr^2} \times \frac{4\pi\epsilon_0 3r}{2q}$$

$$\Rightarrow \frac{E}{V} = \frac{1}{6r}$$

$$\Rightarrow E = \frac{V}{6r}$$

PE

20

(c)

$$E = \frac{1}{4\pi\epsilon_0} \left[\frac{5 \times 10^{-9}}{(1 \times 10^{-2})^2} - \frac{5 \times 10^{-9}}{(2 \times 10^{-2})^2} + \frac{5 \times 10^{-9}}{(4 \times 10^{-2})^2} - \frac{(5 \times 10^{-9})}{(8 \times 10^{-2})^2} + \dots \right]$$

$$\Rightarrow E = \frac{9 \times 10^9 \times 5 \times 10^{-9}}{10^{-4}} \left[1 - \frac{1}{(2)^2} + \frac{1}{(4)^2} - \frac{1}{(8)^2} + \dots \right]$$

$$\Rightarrow E = 45 \times 10^4 \left[1 + \frac{1}{(4)^2} + \frac{1}{(16)^2} + \dots \right]$$

$$- 45 \times 10^4 \left[\frac{1}{(2)^2} + \frac{1}{(8)^2} + \frac{1}{(32)^2} + \dots \right]$$

$$\Rightarrow E = 45 \times 10^4 \left[\frac{1}{1 - \frac{1}{16}} \right] - \frac{45 \times 10^4}{(2)^2} \left[1 + \frac{1}{4^2} + \frac{1}{(16)^2} + \dots \right]$$

$$E = 48 \times 10^4 - 12 \times 10^4 = 36 \times 10^4 \text{ N/C}$$

PE

ANSWER-KEY										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	C	C	C	C	D	A	A	A	D
Q.	11	12	13	14	15	16	17	18	19	20
A.	D	C	C	A	D	A	A	D	C	C

PE