JEE MAIN ANSWER KEY & SOLUTION

PAPER CODE :- PART TEST-1
CLASS-XII

ANSWER KEY

PHYSICS

1.	(C)	**2.**	(B)	**3.**	(B)	**4.**	(A)	**5.**	(A)	**6.**	(B)	**7.**	(C)
8.	(D)	**9.**	(C)	**10.**	(B)	**11.**	(D)	**12.**	(A)	**13.**	(C)	**14.**	(B)
15.	(D)	**16.**	(C)	**17.**	(C)	**18.**	(D)	**19.**	(D)	**20.**	(D)	**21.**	25
22.	20	**23.**	4	**24.**	96	**25.**	9	**26.**	12	**27.**	5	**28.**	19
29.	8	**30.**	240										

CHEMISTRY

31.	(C)	**32.**	(A)	**33.**	(B)	**34.**	(B)	**35.**	(A)	**36.**	(B)	**37.**	(A)
38.	(D)	**39.**	(A)	**40.**	(D)	**41.**	(A)	**42.**	(C)	**43.**	(B)	**44.**	(D)
45.	(B)	**46.**	(B)	**47.**	(A)	**48.**	(C)	**49.**	(A)	**50.**	(A)	**51.**	6
52.	4	**53.**	1	**54.**	2	**55.**	1	**56.**	4	**57.**	30	**58.**	100
59.	350	**60.**	990										

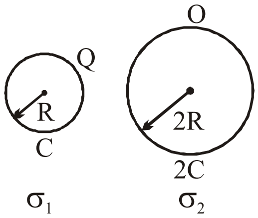
MATHEMATICS

61.	(B)	**62.**	(D)	**63.**	(C)	**64.**	(A)	**65.**	(C)	**66.**	(B)	**67.**	(A)
68.	(D)	**69.**	(D)	**70.**	(C)	**71.**	(B)	**72.**	(A)	**73.**	(C)	**74.**	(C)
75.	(D)	**76.**	(C)	**77.**	(D)	**78.**	(C)	**79.**	(A)	**80.**	(D)	**81.**	1
82.	0	**83.**	3	**84.**	5	**85.**	07	**86.**	7	**87.**	0	**88.**	15
89.	5	**90.**	4										

PHYSICS

1. (C)

Sol.



$$v_1 = v_2 \Rightarrow \frac{\sigma_1 R}{\epsilon_0} = \frac{\sigma_2 \times 2R}{\epsilon_0}$$]

2. (B)

Sol. Magnitude of charge on each inner face

$$q = \frac{2Q-(-Q)}{2} = \frac{3Q}{2}$$

Potential difference $= \dfrac{q}{C} = \dfrac{3Qd}{2\epsilon_0 A}$]

3. (B)

Sol.

$$\frac{kQ^2}{r^2} = \frac{mv^2}{r}$$

$$k = \frac{1}{2}mu^2 = \frac{kQ^2}{2r}$$

4. (A)

Sol. $3.9 \times 10^{-19} = n_1 e,\ 6.5 \times 10^{-19} = n_2 e,\ 9.1 \times 10^{-19} = n_3 e$

n_1, n_2 and n_3 are integers for $e = 1.3 \times 10^{-19}$ Ans.

5. (A)

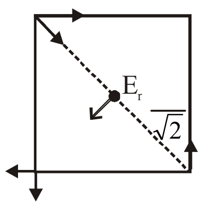
Sol. Force will be zero in an configuration but torque will be zero only when dipole moment in along electric field.]

6. (B)

Sol.

$$E_{r_1} = \frac{2kp\cos 45°}{r^3} = E_{r_2}$$

$$E_{t_1} = \frac{kp\sin 45°}{r^3} = E_{t_2}$$



$$E_0 = 2E_t = \frac{\sqrt{2}kp}{\left(\frac{\ell}{\sqrt{2}}\right)^3} = \frac{2\sqrt{2}}{\ell^3} = \frac{\sqrt{2}\,kp}{\ell^3}$$

ratio $= 2\sqrt{2}$]

7. (C)

Sol. Uf = same

$W = \Delta U \Rightarrow$ same work.]

8. (D)

9. (C)

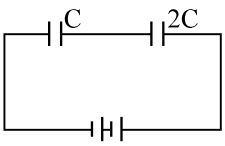
Sol. by symmetry, $\phi = \dfrac{q}{6\varepsilon_0}$

$$\Rightarrow \quad \frac{5\times 10^{-6}}{6} \times 4\pi \times 10^9 \times 9$$

$$\approx 9.4 \times 10^9 \quad]$$

10. (B)

Sol.



$$\Rightarrow \quad C_{eq} = \frac{(2C)C}{2C+C} = 2C/3 \quad]$$

11. (D)

Sol. Plates are brought closer capacity will increase. As battery is removed charge remain constant.

$$U = \frac{1}{2}\frac{Q^2}{C} \quad \Rightarrow \quad U \propto 1/C.$$ Hence stored energy will decrease.]

12. (A)

Sol. $$C = \frac{\in_0 A}{d - \frac{d}{2}\left(1 - \frac{1}{2}\right)} = \frac{4\in_0 A}{3d}$$

13. (C)

Sol. $$\frac{1}{2}CV^2 = pt$$

$$C = \frac{2pt}{V^2} = \frac{2\times 2000\times 0.04}{10^6} \;]$$

14. (B)

15. (D)

Sol. $V = i_g(R+G)$

$20 = i_g(1680+G)$ $\qquad$ $30 = i_g(2930+G)$

$$\frac{3}{2} = \frac{2930+G}{1680+G} \quad \Rightarrow \quad 5040+3G = 5860+2G$$

$$G = 820\,G$$

$$i_g = \frac{20}{2500} = 8\text{ mA} \quad]$$

16. (C)

Sol. $i = \frac{6-2}{4} = 1A$

$v_a - v_b = 2 + 1 \times 1 = 3V$

17. (C)

Sol. $P \propto i^2 R_L$

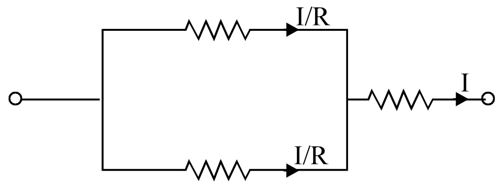
on increasing R_L current will decrease. Hence power.

$P = i^2RL$

$= \left(\frac{E}{r+R_L}\right)^2 R_L$]

18. (D)

Sol.



$P = I^2R = I^2 2 = 18$

$I = 3A$

$P_{net} = \left[\left(\frac{I}{2}\right)^2 R\right] 2 + [I^2R]$

$= \frac{I^2R}{4q} + I^2R$

$= \frac{18}{2} + 18 = 27$]

19. (D)

Sol. $R = \frac{\rho l}{A} = \frac{3 \times 10^{-3} \times 10^{-2}}{8 \times 10^{-4}} = \frac{3}{80}\Omega$

20. (D)

Sol. Copper is metal and germanium is semiconductor. Resistance of a metal decreases and that of a semiconductor increases with decrease in temperature.

21. 25

Sol. $K_i = \frac{kQ^2}{0.2} \quad \frac{kQ^2}{1} = 4\,kQ^2$

$K_f = \frac{1}{4} \times 4\,kQ^2 = \frac{kQ^2}{0.2} - \frac{kQ^2}{x}$

$1 = 5 - \frac{1}{x}$

$\frac{1}{x} = 4$

$x = 25\,cm$]

22. 20

Sol. $kq^2\left(\frac{1}{r}-\frac{1}{2r}\right)=\frac{1}{2}m(V^2)$

$$\frac{kq^2}{2r}=\frac{1}{2}mV^2$$

$$\frac{kq^2}{r}=\frac{1}{2}m\left(\sqrt{2}V\right)^2=\frac{1}{2}m\,(V_{max}^2)$$

$$V_{max}=V\sqrt{2}=\left(10\sqrt{2}\right)\sqrt{2}=20\text{ m/s.}$$]

23. 4

Sol. $U=4[x^2+xy]$

$$\vec{E}=\frac{\partial v}{\partial x}\hat{i}\frac{-\partial v}{\partial x}\hat{j}$$

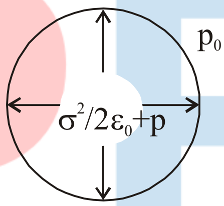
$$\vec{E}=[-8x-4y]\,\hat{i}-4x\hat{j}$$

$$|\vec{E}|=(-8+8)\,\hat{i}-4\hat{j}$$

$$|\vec{E}|=4$$]

24. 96

Sol. $V=\frac{4}{3}\pi r^3$ .... (1)



$\sigma=\frac{Q}{4\pi r^2}$ .... (2)

$\frac{\sigma^2}{2\in_0}+p-p_0=\frac{4s}{r}$ .... (3)

$p-p_0=0$ .... (4)

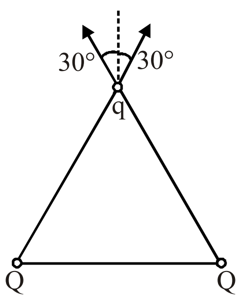
$$\Rightarrow \quad \frac{1}{2\varepsilon_0}\cdot\frac{Q^2}{16\pi^2r^4}=\frac{4s}{r}$$

$$\Rightarrow \quad \frac{1}{2\varepsilon_0}\cdot\frac{n\pi\varepsilon_0 s}{16\pi^2r^3}\cdot\frac{4}{3}\pi r^3=4s$$

$n=96$]

25. 9

Sol.



$$\frac{2kqQ}{\ell^2}\cos 30^\circ=ma$$

$$\frac{2\times9\times10^9\times10^{-8}}{(0.2)^2}\times\frac{\sqrt{3}}{2}=\sqrt{3}\times10^{-6}a$$

$$\Rightarrow \quad a=9\text{ m/s}^2$$]

26. 12

Sol. $\frac{1}{2} \times \frac{4 \times 9}{4+9} = 26^2 = \frac{1}{2} \times (4+9)\, v^2$

$v = 26 \times \frac{6}{13} = 12$]

27. 5

[Sol. $R = \frac{V}{I} = \frac{6}{1.2} = 5\ \Omega$]

28. 19

Sol. According to Kirchhoffs Voltage Law, the sum of the potential drops equal to the sum of the potential rises;

Therefore, $30 = 2 + 1 + V_1 + 3 + 5$

or $V_1 = 30 - 11 = 19$ V **Ans.**

29. 8

Sol. Rate of heat developed, $P = V^2/R$

For given V,P $\propto 1/R = A/\rho l = \pi r^2/\rho l$

Now, $P_1/P_2 = \left(r_1^2/r_2^2\right)\left(l_2/l_1\right)$

As per question, $l_2 = l_1/2$ and $r_2 = 2r_1$

$P_1/P_2 = \left(\frac{1}{4}\right) \times \left(\frac{1}{2}\right) = \frac{1}{8}$

$P_2 = 8P_1$

30. 240

$V^2/2R = 60$ W $\Rightarrow$ $V^2/R = 120$ W

When the two resistance are connected in parallel combination, power consumed is

$2V^2/R = 120(2) = 240$ W

31. (C)

Sol. Ice has open cage structure

32. (A)

Sol. NaCl (moles) = 1

$$\text{Moles of water} = \frac{1000}{18} = 55.55$$

$$\text{mole fraction of NaCl} = \frac{1}{55.55+1}$$

$$= \frac{1}{56.55} = 0.0176 \simeq 0.0177$$

33. (B)

Sol. $$\text{molarity} = \frac{\text{moles}}{\text{vol.of solution (in litre)}}$$

$$= \frac{5/34}{100/1000} = 0.147 \times 10 = 1.47 \approx 1.5$$

34. (B)

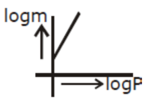
Sol. 150 g contain — 10 g

$$\text{19 contain} — \frac{10}{150}$$

$$\text{100 g contain } \frac{10}{150} \times 100 = 6.66 \simeq 6.25\%$$

35. (A)

Sol.



36. (B)

Sol. Pure A : $X_B = 0$

$= P_T = P_A{}^{\circ} = 120$

Pure B : $X_B = 1$

$= P_T = P_B{}^{\circ} = 120 + 75 = 195$

37. (A)

Sol. Negative deviations from Raoult's law

38. (D)

Sol. $K_3\left[Fe(CN)_6\right] \rightarrow 3K^+ + Fe(CN)_6^{-3} \rightarrow 3+1=4$

(a) $NaCl \rightarrow Na^+ + Cl^- \rightarrow 2$

(b) $Na_2SO_4 \rightarrow 2Na^+ + SO_4{}^{-2} \rightarrow 3$

(c) $Al_2(SO_4)_3 \rightarrow 2Al^{+3} + 3SO_4^{-2} \rightarrow 5$

(d) $Al_2(NO_3)_3 \rightarrow Al^{+3} + 3NO_3^- = 1+3=4$

39. (A)

Sol. Osmosis

40. (D)

Sol. Hypertonic solution

41. (A)

Sol. Volume of 100 g of the solution = $\frac{100}{d}$

$= \frac{100}{1.09}$ mL $= \frac{100}{1.09 \times 1000}$ L

$= \frac{1}{1.09 \times 10}$ L

Number of moles of H_2SO_4 in 100 gram of the solution = $\frac{13}{98}$

Molarity $= \frac{\text{No of moles of } H_2SO_4}{\text{Volume of solution in litre}}$

$= \frac{13}{98} \times \frac{1.09 \times 10}{1} = 1.445$ M

42. (C)

Sol. Wt . of CH_3COOH dissolved = 5g

Eq. of CH_3COOH dissolved = $\frac{5}{60}$

Volume of ethanol = 1 litre = 1000mL.

$\therefore$ Weight of ethanol = 1000 × 0.789 = 789 g

$\therefore$ Molality of solution = $\frac{\text{Moles of solute}}{\text{wt of solvent in kg}}$

$= \frac{\frac{5}{60 \times 789}}{1000} = 0.1056$

43. (B)

Sol. Wt. of NaOH dissolved = 0.5 g

Vol. of NaOH solution = 500 mL

Calculation of molarity :

0.5 g of NaOH = $\frac{0.5}{40}$ moles of NaOH

[$\because$ Mol. wt. of NaOH = 40] = 0.0125 moles

Thus 500 mL of the solution contain NaOH = 0.0125 moles

$\therefore$ 1000 mL of the solution contain

$= \frac{0.0125}{500} \times 1000 = 0.025$ M

Hence molarity of the solution = 0.025 M

Calculation of normality :

Since NaOH is monoacidic ;

Equivalent wt. of NaOH = Mol. wt. of NaOH = 40

$\therefore$ 0.5 g of NaOH = $\frac{0.5}{40}$ g equivalents

= 0.0125 g equivalents

Thus 500 mL of the solution contain NaOH = 0.0125 g equ.

$\therefore$ 1000 mL of the solution contain

$= \frac{0.0125}{500}$ 1000 = 0.025

Hence normality of the solution = 0.025 N

44. (D)

Sol. Since 18 g of water = 1 mole

25 g of water = $\frac{25}{18}$ = 1.38 mole

Similarly, 46 g of ethanol = 1 mole

25 g of ethanol = $\frac{25}{46}$ = 0.55 moles

Again, 60 g of acetic acid = 1 mole

50 g of acetic acid = $\frac{50}{60}$ = 0.83 mole

$\therefore$ Mole fraction of water

$= \frac{1.38}{1.38 + 0.55 + 0.83}$

Similarly, Mole fraction of ethanol

$= \frac{0.55}{1.38 + 0.55 + 0.83} = 0.19$

Mole fraction of acetic acid

$= \frac{0.83}{1.38 + 0.55 + 0.83} = 0.3$

45. (B)

Sol. $N_1V_1 = N_2V_2$

$6.5\,V_1 = 3.5\,(V_1 + x)$

$6.5\,V_1 = 3.5\,V_1 + 3.5\,x$

$3V_1 = 3.5\,x$

$\frac{V_1}{x} = \frac{3.5}{3} = \frac{7}{6}$

46. (B)

Sol. (i) Equivalent wt. of $H_2SO_4 = \frac{\text{Mol. wt}}{\text{Bascity}} = \frac{98}{2} = 49$

$\therefore$ Amount of H_2SO_4 per litre (strength) = Normality × Equivalent wt. = $\frac{1}{7} \times 49 = 7$ g/L

Amount in 150 mL = $\frac{7 \times 150}{1000} = 1.05$ g

(ii) Molecular wt. of

$NaHCO_3 = 23 + 1 + 12 + 48 = 84$

Amount of NaHCO3 required to produce 1000 c.c. of one molar solution = 84 g

Amount present per litre in 0.2 M solution = 84 × 0.2 = 16.8 g

$\therefore$ Amount present in 250 c.c.

$= \frac{16.8 \times 250}{1000} = 4.2$ g

(iii) Equivalent weight of

$Na_2CO_3 = \frac{\text{Mol. wt}}{\text{No. of positive valencies}}$

$= \frac{106}{2} = 53$

Amount of Na_2CO_3 = Normality × Equivalent wt.

$= \frac{1}{10} \times 53 = 5.3$ g/L

$\therefore$ Amount present in 400 c.c. = $\frac{5.3 \times 400}{1000} = 2.12$ g

(iv) We know that 1 molal solution of a substance contains 1000 g of solvent.

$\therefore$ Wt. of KOH in 1052 g of 1 m KOH solution = 1052

– 1000 = 52 g

47. (A)

Sol. Since for calculating osmotic pressure we require millimoles/litre therefore

$Na^+ = 138$, $Ca^{2+} = \frac{5.2}{2} = 2.6$, $K^+ = 4.5$,

$Mg^{2+} = \frac{2.0}{2} = 1.0$, $Cl^- = 105$,

$HCO_3^- = 24$, $PO_4^{3-} = \frac{2.2}{3} = 0.73$,

$SO_4^{2-} = \frac{0.5}{2} = 0.25$, Proteins = 16,

others = 1.0

Total = 294.18 millimoles/litre $= \frac{294.18}{1000} = 0.294$ moles/litre

Now since $\pi = CRT$

$= 0.294 \times 0.0821 \times .310 = 7.47$ atm

48. (C)

Sol. Wt. of solute, w = 0.450 g

Wt. of solvent, W = 22.5 g

Mol. wt. of solute, m = 60

Molal elevation constant K_b = ?

Boiling point elevation, $\Delta T_b = 0.170°C$

Substituting these values in the equation-

$$K_b = \frac{m \times W \times \Delta T_b}{1000 \times w}$$

$$= \frac{60 \times 22.5 \times 0.170}{1000 \times 0.450} = 0.51°C$$

49. (A)

Sol. ΔT_f = freezing point of water – freezing point of solution

$= 0°C - (-1.1o\,C) = 1.1°$

We know that,

$\Delta T_f = i \times K_f \times m$

$1.1 = i \times 1.86 \times 0.2$

$\therefore i = \frac{1.1}{1.86 \times 0.2} = 2.95$

But we know

$i = 1 + (n - 1)\alpha$

$2.95 = 1 + (3 - 1)\alpha = 1 + 2\alpha$

$\alpha = 0.975$

Van't Hoff factor (i) = 2.95

Degree of dissociation = 0.975

Percentage degree of dissociation = 97.5

50. (A)

Sol. Let x L of A and (2 ¡V x) L of B are mixed.

$M_1V_1 + M_2V_2 = M_R(V_1 + V_2)$

$0.5 \times x + 0.1(2 - x) = 0.2 \times 2$

(0.5 – 0.1) x = 0.4 ¡V 0.2

$0.4x = 0.2 \Rightarrow x = 0.5$ L

$\therefore$ 0.5 L of A and 1.5 L of B should be mixed.

51. 6

Sol. $\frac{P_o - P_s}{P_s} = i \times m \times \frac{18}{1000}$

$\Rightarrow \frac{21.08 - 20}{20} = 3 \times m \times \frac{18}{1000}$

$\Rightarrow m = 1$

$\Delta T_f = i \times K_f \times m = 3 \times 2 \times 1 = 6$ Ans.

52. 4

Sol. We know, $\Delta T_f = K_f m$

So, $K_f = \frac{\Delta T_f}{m} = \frac{2 \text{ deg}}{0.5 \text{ mol kg}^{-1}} = 4 \text{ deg kg mol}^{-1}$

53. 1

Sol. $\pi = CRT \times i$ $\alpha = \frac{1}{5} = 0.2$

$Ci = 0.6$

$i = 1.2$

$[H^+] = C\alpha = 0.5 \times 0.2 = 0.1$

pH = 1 **Ans.**

54. 2

Sol. $\frac{P^o - P_s}{P_s} = i.\frac{n}{N} \Rightarrow \frac{P^o}{P_s} - 1 = \frac{2 \times 0.2}{4}$

$\Rightarrow \frac{P^o}{P_s} = 1 + \frac{1}{10} \Rightarrow \frac{P^o}{P_s} = \frac{11}{10}$

$\therefore P_s = \frac{10P^o}{11} = \frac{10 \times 24.2}{11}$ torr

i.e. $P_s = 22$ torr $= \frac{22}{11} = 2$ torr Ans.

55. 1

Sol. $i = 1 + \alpha (2 - 1)$

$= (1 + \alpha)$

$\Delta T_f = iK_f m$

$= (1 + \alpha) \times K_f \times 0.4$

$0.93 = (1 + \alpha)\, 1.86 \times 0.4$

$(1 + \alpha) = \frac{0.93}{1.86 \times 0.4}$

$\alpha = \frac{1}{4}$

$Ka = \frac{C\alpha^2}{(1 - \alpha)} \Rightarrow \frac{0.4 \times \frac{1}{4} \times \frac{1}{4}}{\frac{3}{4}}$

$\Rightarrow \frac{0.1}{3} \times 30 = 1$ **Ans.**

56. 4

Sol. $\Delta T_b = iK_b m$

$0.93 = i \times 1.86 \times 0.1$

$i = 5$

$K_4[Fe(CN)_6] \longrightarrow 4K^+ + [Fe(CN)_6]^{4-}$

57. 30

Sol. Total mass of solution = (15 + 35) g = 50 g mass percentage of methyl alcohol

$$= \frac{\text{Mass of methyl alcohol}}{\text{Mass of solution}} \times 100$$

$$= \frac{15}{50} \times 100 = 30\%$$

58. 100

Sol. Here it is given that

$w = 0.15\ g, \Delta T_b = = 0.216°C$

$W = 15g\ K_b = 2.16°C\ m = ?$

Substituting values in the expression ,

$$m = \frac{1000 \times K_b \times w}{\Delta T_b \times W}$$

$$m = \frac{1000 \times 2.16 \times 0.15}{0.216 \times 15} = 100$$

59. 350

Sol. Molarity of mixture of 6 M and 3 M HCl

$$= \frac{350 \times 6 + 650 \times 3}{1000} = 4.05\ M$$

Now, apply dilution formula

$$M_1V_1 = M_2V_2 \text{ or } V_2 = \frac{4.05 \times 1000}{3}$$

$= 1350\ mL$

Volume of water to be added

$= 1350 - 1000 = 350\ mL$

60. 990

Sol. Molarity of solution

$$M = \frac{w_B \times 1000}{m_B \times V} = \frac{2.65 \times 1000}{106 \times 250} = 0.1$$

$M_1V_1 = M_2V_2$

$0.1 \times 10 = 0.001\ (10 + x) \Rightarrow x = 990\ mL$

61. (B)

Sol. $|A^{-1}| = 25$

$\therefore |A| = \frac{1}{25}$

(A) $A^{-1} = \frac{AdjA}{|A|} = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 2 & 1 \\ 0 & 1 & 4 \end{bmatrix}$

$\Rightarrow AdjA = \frac{1}{25}\begin{bmatrix} 2 & 3 & 1 \\ -1 & 2 & 1 \\ 0 & 1 & 4 \end{bmatrix}$

(B) $|Adj(AdjA)| = |A^4| = \frac{1}{5^8}$

(C) $|5Adj(AdjA)| = 5^3|Adj(AdjA)| = 5^3 \times \frac{1}{5^8} = 5^{-5}$

(D) $|Adj(A^{-1})| = |(Adj(A))^{-1}| = \frac{1}{|AdjA|} = \frac{1}{|A|^2} = 5^4$

62. (D)

Sol. $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}\begin{vmatrix} -a & -b & -c \\ c & a & b \\ b & c & a \end{vmatrix}$

$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 \geq 0$

63. (C)

Sol. (A) $f(x) = e^{1/2\,\ell n\,x} = , \sqrt{x}$ $\quad D : x > 0 \quad g(x) = \sqrt{x}, D : x \geq 0$

(B) $\tan^{-1}(\tan x)$ $\quad D : x \neq \pm(2n+1)\frac{\pi}{2}$

$\cot^{-1}(\cot x)$ $\quad D : x \neq \pm n\pi$

(C) $f(x) = \cos^2 x + \sin^4 x$

$= \cos^2 x + (1 - \cos^2 x)^2$

$= 1 - \cos^2 x + \cos^4 x$

$= \sin^2 x + \cos^4 x$

$g(x) = \sin^2 x + \cos^4 x$

(D) $f(x) = \frac{|x|}{x},$ $\quad D : x \neq 0$

$g(x) = \text{sgn}(x),$ $\quad D : x \in R$

64. (A)

Sol. $AA^T = I$

$$\begin{bmatrix} 0 & \alpha & \alpha \\ 2\beta & \beta & -\beta \\ \gamma & -\gamma & \gamma \end{bmatrix}\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\alpha^2 & 0 & 0 \\ 0 & 6\beta^2 & 0 \\ 0 & 0 & 3\gamma^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow 2\alpha^2 = 1,\ 6\beta^2 = 1,\ 3\gamma^2 = 1$

$$\therefore \alpha = \pm\frac{1}{\sqrt{2}},\ \beta = \pm\frac{1}{\sqrt{6}},\ \gamma = \pm\frac{1}{\sqrt{3}}$$

65. (C)

Sol. $(I+M)^3 - 7M = (I+M)(I+M)(I+M) - 7M$

$= (I+M+M+M^2)(I+M) - 7M$

$= I + 2M + M^2 + M + 2M^2 + M^3 - 7M$

$= 3M^2 + M^3 - 4M + I$

$= 3M + M^2 - 4M + I$

$= 3M + M - 4M + I$

$= I$

66. (B)

Sol. $h(f(g(x)) = h\left(f\left(\sqrt{x^2+1}\right)\right) = h(x^2)$

$\because \quad x^2 \geq 0$

$$h(x) = \begin{cases} 0, & x = 0 \\ x^2, & x \neq 0 \end{cases}$$

67. (A)

Sol. $\theta_1 + \theta_2 = \pi$

$$\theta_1 = \sin^{-1}\left[\frac{4}{5}\sqrt{1-\frac{1}{9}} + \frac{1}{3}\sqrt{1-\frac{16}{25}}\right]$$

$$= \sin^{-1}\left[\frac{8\sqrt{2}}{15} + \frac{3}{15}\right]$$

$$= \sin^{-1}\left[\frac{8\sqrt{2}+3}{15}\right]$$

$$\text{since} \left[\frac{8\sqrt{2}+3}{15}\right] < 1$$

$$\theta_1 < \frac{\pi}{2}$$

$$\theta_2 = \left(\frac{\pi}{2} - \sin^{-1}\frac{4}{5} + \frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right) = (\pi - \theta_1) > \frac{\pi}{2}$$

$\therefore \theta_1 < \theta_2$

68. (D)

Sol. $\theta = \sin^{-1}x + \cos^{-1}x - \tan^{-1}x$

$= \frac{\pi}{2} - \tan^{-1}x$

Domain $x \in [-1, 1]$

But given $x \geq 0$

$\Rightarrow x \in [0, 1]$

$\theta = \frac{\pi}{2} - \tan^{-1}x$

for $x \in [0, 1]$

$\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$

69. (D)

Sol. We have $(1)^2 = 1, (2)^2 = 4, (3)^2 = 9, (4)^2 = 16, (5)^2 = 25, (6)^2 = 36$

$\therefore$ $R = \{(1, 1), (4, 2), (9, 3), (16, 4), (25, 5), (36, 6)\}$

Domain of $R = \{x ; (x, y) \in R\} = \{1, 4, 9, 16, 25, 36,\}$

Range of $R = \{y ; (x, y) \in R\} = \{1, 2, 3, 4, 5, 6\}$

$\therefore$ (1), (2), (3) are true

$\therefore$ The correct answer is (4)

70. (C)

Sol. $\frac{\pi}{2} - \sin^{-1}\left(-\sin\left(\frac{7\pi}{6}\right)\right) = \frac{\pi}{2} + \sin^{-1}\sin\left(\frac{7\pi}{6}\right)$

$= \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

71. (B)

Sol. $\cot^{-1}\left\{\frac{\sqrt{1-\sin x} + \sqrt{1+\sin x}}{\sqrt{1-\sin x} - \sqrt{1+\sin x}}\right\}$

Rationalize the term in the bracket

$= \cot^{-1}\left(\frac{2 + 2\sqrt{1-\sin^2 x}}{-2\sin x}\right) = \cot^{-1}\left(\frac{1-\cos x}{-\sin x}\right)$

$= \cot^{-1}\left(-\tan\frac{x}{2}\right) = \frac{\pi}{2} - \tan^{-1}\left(-\tan\frac{x}{2}\right)$

$= \frac{\pi}{2} + \tan^{-1}\tan\frac{x}{2}$ since $\frac{x}{2} \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$= \frac{\pi}{2} + \frac{x}{2}$

72. (A)

Sol. $\because 4\{x\} = x + [x]$ ....(1)

Case-1 Let $x = I$

$O = I + I \Rightarrow I = 0$

Case -2, Let $x = I + f, 0 < f < 1$

$\therefore$ equation (1) becomes

$4f = I + f + I$

$f = \frac{2I}{3}$ $\therefore I = 1, f = \frac{2}{3}$ $\therefore r = \frac{5}{3}$

$\therefore$ two solutions

73. (C)

Sol. $x + 2y = 8$

$x = 8\text{-}2y$

for $y = 1, x = 6$

$y = 2 , x = 4$

$y = 3, x = 2$

$R = \{(2,3),(4,2),(6,1)\}$

Domain of $R = \{2,4,6\}$

74. (C)

Sol. Quadrant 1

$\text{sgn}(\sin x) = 1, \text{sgn}(\cos x) = 1, \text{sgn}(\cot x) = 1, \text{sgn}(\tan x) = 1$

value $= 1 + 1 + 1 + 1 = 4$

Quadrant 2

value $= 1 - 1 - 1 - 1 = -2$

Quadrant 3

value $= -1 - 1 + 1 + 1 = 0$

Quadrant 4

value $= -1 + 1 - 1 - 1 = -2$

By considering all four quadrants one by one we get range of f(x) as $\{-2, 0, 4\}$

75. (D)

Sol. Let $A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}, A^2 = \begin{bmatrix} a^2 & ab + bc \\ 0 & c^2 \end{bmatrix}$

$$A^3 = \begin{bmatrix} a^3 & a^2b + abc + bc^2 \\ 0 & c^3 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 8 & -57 \\ 0 & 27 \end{bmatrix}$$

$a^3 = 8$	$c^3 = 27$	$a^2b + abc + bc^2 = -57$
$a = 2$	$c = 3$	$b = -3$

$$A = \begin{bmatrix} 2 & -3 \\ 0 & 3 \end{bmatrix}$$

$$5A = \begin{bmatrix} 10 & -5 \\ 0 & 15 \end{bmatrix}$$

$$5A - A^2 = \begin{bmatrix} 10 & -5 \\ 0 & 15 \end{bmatrix} - \begin{bmatrix} 4 & -15 \\ 0 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

$|SA - A^2| = 36 - 0 = 36$

76. (C)

Sol. One element must have one image. So only (C)

77. (D)

Sol. $D = \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} + \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix}$

$\Delta = xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$

$\Delta = (xyz + 1) \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$

$\Delta = (xyz + 1)(x - y)(y - z)(z - x) = 0$

$\therefore [xyz = -1]$

78. (C)

Sol. for symmetric matrix;

$[A^T = A]$

for skew symmetric matrix;

$[A^T = -A]$

we know that via property

$[\text{adj}(AB) = (\text{adj } B)(\text{adj } A)]$

79. (A)

Sol. Put $x = 1$,

$af(2) + b f\left(\frac{1}{2}\right) = 1$ ....(i)

Put $x = \frac{-1}{2}$

$a f\left(\frac{1}{2}\right) + bf(2) = \frac{-1}{2}$ ....(ii)

solving (i) and (ii), a(i) – b(ii)

$\Rightarrow f(2)(a^2 - b^2) = a - (-\frac{b}{2})$

$f(2)(a^2 - b^2) = \frac{2a + b}{2}$

$f(2) = \frac{2a + b}{2(a^2 - b^2)}$

80. (D)

Sol. $R = \{(a, b ; 1 + ab > 0)\}$

Reflexive : for (a, a) $1 + a^2 > 0$ $\Rightarrow$ Reflexive

Symmetric : if (a, b) then (b, a) is also present

for (a, b) $1 + ab > 0$

then (b, a) $1 + ba > 0$ is also true

so it is symmetric

for transitive if (a, b) and (b, c) true then (a, b) is also present

$1 + ab > 0$ & $1 + bc > 0$

then it is not necessary that $1 + ac > 0$

for example $a = 1, b = -1/2, c = -2$

$1 + ab > 0$ $1 + bc > 0$

but $1 + ac < 0$

hence not trasitive

81. 1

Sol. For non-trivial solution $\begin{vmatrix} \lambda & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ -1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$

Expanding along C_1, we get

$$\lambda = \sin 2\alpha + \cos 2\alpha$$

$\therefore -\sqrt{2} \le \lambda \le \sqrt{2}$

$a = -\sqrt{2}$ and $b = \sqrt{2}$

therefore, $a + b + 1 = -\sqrt{2} + \sqrt{2} + 1 = 1$

82. 0

Sol. $f(x) = \sin^{-1}[2 - 4x^2]$

$-1 \le [2 - 4x^2] \le 1$

$-1 \le 2 - 4x^2 < 2$

$0 < 4x^2 \le 3$

$0 < x^2 \le \frac{3}{4}$

$x \in \left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right] - \{0\}$

$\therefore$ Number of integral values of x are 0.

83. 3

Sol. $\tan\theta = \dfrac{2\tan^2\theta - \frac{1}{3}\tan\theta}{1 + \frac{2}{3}\tan^3\theta}$

$\tan\theta = \left[\dfrac{2\tan\theta - \frac{1}{3}}{1 + \frac{2}{3}\tan^3\theta} - 1\right] = 0$

$\tan\theta = 0$ or $\dfrac{2\tan\theta - \frac{1}{3}}{1 + \frac{2}{3}\tan^3\theta} = 1$

$\tan^3\theta - 3\tan\theta + 2 = 0$

$(\tan\theta - 1)^2(\tan\theta + 2) = 0$

$\tan\theta = 1,\ \tan\theta = -2$

but since $0 < \theta < 90$

so $\tan\theta = 1$

84. 5

Sol. $g(f(x)) = x$

$\Rightarrow \quad g'(f(x))\,f'(x) = 1$

put $f(x) = -\frac{7}{6}$ i.e. $x = 1$

$\therefore g'\left(-\frac{7}{6}\right) = \frac{1}{5}$

85. 07

Sol. $-x^2-2x+n \geq 0$

downward parabola (+)ve between roots

roots, $x = -1 \pm \sqrt{1+n}$

thus, $\Rightarrow\ -1 \pm \sqrt{1+n}\ \geq 5 \Rightarrow n \geq 35$

86. 7

Sol. $10 - x \geq 0$

$10 \geq x,\ x \leq 10$

$$\tan^{-1}\frac{x^3-6x^2+11x-6}{x(e^x-1)} > 0$$

$x \neq 0$

$$\frac{(x-1)(x-2)(x-3)}{x(e^x-1)} > 0$$

$$\frac{(x-1)(x-2)(x-3)}{x^2} > 0$$

as sign of $(e^x - 1)$ changes as of x

so possible integer are 4,5,6,7,8,9,10

Total = 7

87. 0

Sol. Every odd function if defined at x = 0 then f(0) = 0

Hence period = 2

So f(0) = f(2) = f(4) = f(6) = 0

88. 15

Sol. Let $\frac{y}{15} = x$

$$(\tan^{-1}x + \cot^{-1}x)^2 - 2\tan^{-1}x\left(\frac{\pi}{2} - \tan^{-1}x\right) = \frac{5\pi^2}{8}$$

$$2(\tan^{-1}x)^2 - 2\left(\frac{\pi}{2}\right)\tan^{-1}(x) - \frac{3\pi^2}{8} = 0$$

$$\tan^{-1}x = -\frac{\pi}{4}$$

$x = -1$

$\therefore\ \frac{y}{15} = -1$

$\Rightarrow y = -15$

$\therefore\ |y| = 15$

89. 5

Sol. Let $\operatorname{cosec}^{-1}\sqrt{1+4\alpha^4}=\theta$

$\operatorname{cosec}\theta=\sqrt{1+4\alpha^4}$

$\tan\theta=\dfrac{1}{2\alpha^2}=\dfrac{2}{4\alpha^2}$

$\theta=\tan^{-1}\dfrac{2}{4\alpha^2}$

$\theta=\tan^{-1}\dfrac{(2\alpha+1)-(2\alpha-1)}{1+(2\alpha+1)(2\alpha-1)}$

so $\displaystyle\sum_{\alpha=1}^{\infty}\operatorname{cosec}^{-1}\sqrt{1+4\alpha^4}=\sum_{\alpha=1}^{\infty}\tan^{-1}\frac{(2\alpha+1)-(2\alpha-1)}{1+(2\alpha+1)(2\alpha-1)}$

$\displaystyle=\sum_{\alpha=1}^{\infty}\tan^{-1}(2\alpha+1)-\sum_{\alpha=1}^{\infty}\tan^{-1}(2\alpha-1)$

$=(\tan^{-1}3-\tan^{-1}1)+(\tan^{-1}5-\tan^{-1}3)+(\tan^{-1}7-\tan^{-1}5)+.........$

$=\tan^{-1}\infty-\tan^{-1}1=\dfrac{\pi}{2}-\dfrac{\pi}{4}=\dfrac{\pi}{4}$

So $a=1, b=4$

$\therefore\ a+b=1+4=5$

90. 4

Sol. As $A=\{1,2\}, B=\{0\}$

$\therefore$ $n(A)=2, n(B)=1$

$\therefore$ number of relations from A to B is $=2^{n(A)\times n(B)}$

$=2^{2\times1}=4$