JEE MAIN ANSWER KEY & SOLUTION

PAPER CODE :- PART TEST-2
CLASS-XII

ANSWER KEY

PHYSICS

1.	(D)	**2.**	(C)	**3.**	(A)	**4.**	(C)	**5.**	(D)	**6.**	(A)	**7.**	(C)
8.	(A)	**9.**	(A)	**10.**	(A)	**11.**	(A)	**12.**	(C)	**13.**	(B)	**14.**	(B)
15.	(D)	**16.**	(C)	**17.**	(B)	**18.**	(A)	**19.**	(D)	**20.**	(D)	**21.**	1
22.	6	**23.**	6	**24.**	16	**25.**	4	**26.**	0	**27.**	4	**28.**	3
29.	32	**30.**	10										

CHEMISTRY

31.	(B)	**32.**	(C)	**33.**	(B)	**34.**	(B)	**35.**	(B)	**36.**	(B)	**37.**	(A)
38.	(B)	**39.**	(D)	**40.**	(B)	**41.**	(C)	**42.**	(B)	**43.**	(C)	**44.**	(C)
45.	(A)	**46.**	(A)	**47.**	(C)	**48.**	(A)	**49.**	(D)	**50.**	(D)	**51.**	2
52.	2	**53.**	8	**54.**	3	**55.**	391	**56.**	0.29	**57.**	2	**58.**	20
59.	0.25	**60.**	4										

MATHEMATICS

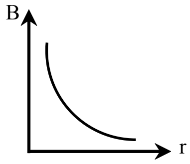
61.	(B)	**62.**	(C)	**63.**	(C)	**64.**	(A)	**65.**	(A)	**66.**	(B)	**67.**	(C)
68.	(D)	**69.**	(A)	**70.**	(C)	**71.**	(A)	**72.**	(C)	**73.**	(D)	**74.**	(B)
75.	(A)	**76.**	(C)	**77.**	(B)	**78.**	(D)	**79.**	(C)	**80.**	(C)	**81.**	5
82.	4	**83.**	2	**84.**	66	**85.**	36	**86.**	1	**87.**	7	**88.**	39
89.	3	**90.**	481										

SOLUTIONS

PHYSICS

1. (D)

Sol. $B \propto \frac{1}{r}$



2. (C)

Sol. $H = 2 \times 10^3$, for solenoid $H = n\,i$

$$n = \frac{150}{15 \times 10^{-2}} = 1000 \Rightarrow i = \frac{H}{n} = \frac{2 \times 10^3}{10^3} = 2A$$

3. (A)

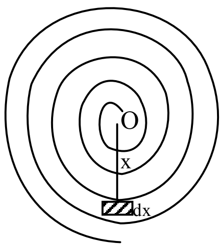
Sol. $\therefore$ Net magnetic field at centre is zero

$\therefore$ $B_1 = B_2$

$\therefore$ $\frac{B_1}{B_2} = 1$

4. (C)

Sol.



An element is assumed at distance x from center whose width is dx. No. of turns in width b–a = N

$\therefore$ No. of turns is width dx = n = $\left(\frac{N}{b-a}\right)dx$

$$dB = \frac{\mu_0 ni}{2x} = \frac{\mu_0 iN}{2(b-a)}\frac{dx}{x}$$

$$\therefore B = \frac{\mu_0 iN}{2(b-a)}\int_a^b \frac{dx}{x} = \frac{\mu_0 Ni}{2(b-a)}\log_e\left(\frac{b}{a}\right)$$

5. (D)

Sol. $F = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{r}$

$$F' = \frac{\mu_0}{2\pi}\frac{\left(\frac{i_1}{3}\right)\left(\frac{i_2}{3}\right)}{3r} = \frac{F}{27}$$

6. (A)

Sol. $\tan\phi' = \frac{\tan\phi}{\cos\alpha} \Rightarrow \tan 45^\circ = \frac{\tan\phi}{\cos 30^\circ}$

$\phi = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

7. (C)

Sol. $F = iB\,\ell \sin\theta$

$F_{PQ} = IB\,\ell \sin 0° = 0$

$F_{QR} = IB\ell \sin 90° = IB\ell$

8. (A)

9. (A)

Sol. At poles $B_H = 0$

$\phi = 90^o$

10. (A)

Sol. $\because$ Particle moves undeflected

$\therefore$ $F_{Net} = 0$

$\therefore$ $qE = qvB$

$\therefore$ $v = \dfrac{E}{B}$

11. (A)

Sol. $\phi_1 = NBA \cos 0^o = NBA$

$\phi_2 = NBA \cos \pi = -NBA$

$$e = -\frac{d\phi}{dt} = \frac{2NBA}{t}$$

12. (C)

Sol. Some domains increased which are in the direction of applied field, other decrease

13. (B)

Sol. By Right hand thumb rule the field by both the segments are out of the plane is along +ve z axis.

14. (B)

15. (D)

Sol. τ due to $B_0 = B_0 \times I \times \pi R^2$

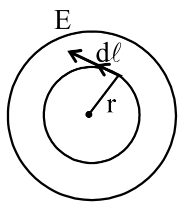
τ due to $mg = mg \times R$

$mg\,R = B_0\,I \times \pi R^2$ (for equilibrium)

$$\therefore I = \frac{mg}{B_0 \pi R^2}$$

16. (C)

Sol. $\displaystyle\int E.d\ell = -\pi r^2 \frac{dB}{dt}$



$E \times 2\pi r = +\pi r^2 \times \dfrac{dB}{dt}$ as $\dfrac{dB}{dt} = -ve$

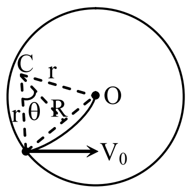
$E = +ve$

$\therefore$ assume direction is right

so q charge move along 3.

17. (B)

Sol. $r = \frac{mV_0}{qB}$



$R = 2\, r \sin\theta$

$\Rightarrow R = \frac{2mV_0}{qB}\sin\theta$

$\Rightarrow V_0 = \frac{qBR}{2m\sin\theta}$

18. (A)

Sol. L will decrease as Bi is diamagnetic

$I = \frac{V}{X_L}$ will increase

19. (D)

Sol. $\tau = NIAB\sin\theta \Rightarrow I\frac{\sqrt{3}}{4}\ell^2 B\sin 90^\circ = \frac{\sqrt{3}}{4}IB\ell^2$

20. (D)

Sol. $T = 2\pi\sqrt{\frac{I}{MB_H}}$

$T' = 2\pi\sqrt{\frac{2I}{MB_H}} = \sqrt{2}\,T$

21. 1

Sol. $R = \frac{mv}{qB}$

$q \times 12 \times 10^3 = \frac{1}{2}m \times (10^6)^2$

$\frac{24\times 10^3}{10^{12}} = \frac{m}{q}$

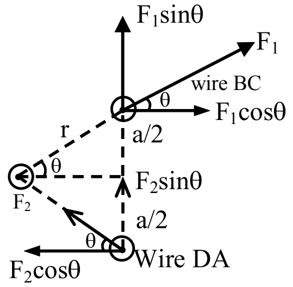
$R = \frac{24\times 10^3 \times 10^6}{10^{12}\times 0.2}$

$R = 12 \times 10^{-2}$ m

$R = 12$ cm

22. 6

Sol.



Total force $= (F_1 + F_2)\sin\theta$

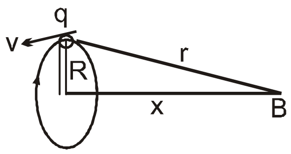
$$F_1 = F_2 = \frac{\mu_0 i_1 i_2}{2\pi r}\frac{a}{2r}$$

$$= \frac{\mu_0 i_1 i_2 a}{4\pi r^2}$$

$= 6 \times 10^{-4}$ Newton

23. 6

Sol. $$\vec{B} = \frac{\mu_0}{4\pi} q\left(\frac{\vec{v}\times\vec{r}}{r^3}\right)$$



$$B = \frac{\mu_0}{4\pi}\frac{qv}{(x^2 + R^2)}$$

$$= \frac{10^{-7} \times 1 \times 0.6 \times 10^4 \pi}{1}$$

$= 6\pi \times 10^{-4}$ T

24. 16

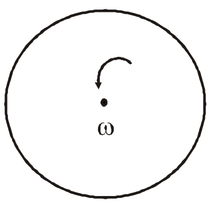
Sol. $$i = \frac{\Delta q}{\Delta t} = \frac{50 \times 10^6 (20 - 18)}{2} = 50\ \mu C/sec$$

$B = \mu_0 ni = 4\pi \times 10^{-7} \times 8000 \times 50 \times 10^{-6}$

$= 16\pi \times 10^{-8}$ T

25. 4

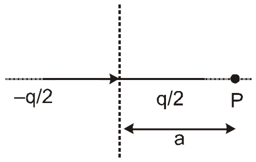
Sol. $$\frac{M}{\frac{mr^2\omega}{2}} = \frac{q}{2m}$$



$$M = \frac{qr^2\omega}{4}$$

26. 0

Sol.



Point P on the extended part of line thus

BP = zero

27. 4

Sol. $\tau = M \times B$

$= MB \sin 90°$

$= Ni\pi R^2 B$

$2\pi RN = l$

$$R = \frac{\ell}{2\pi N}$$

$$\tau = Ni\pi \frac{\ell^2}{4\pi^2 N^2} B$$

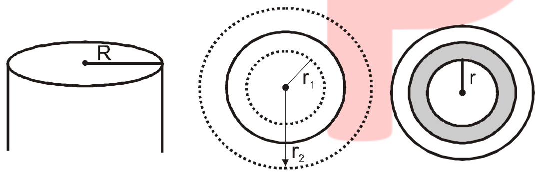
$\tau max \Rightarrow N = 1$

$$\Rightarrow t = \frac{i\pi\ell^2}{4\pi} B$$

[N = 4]

28. 3

Sol.



$$B_1 = \frac{\mu_0 i_1}{2\pi r_1}$$

$$B_1 = \frac{\mu_0 b r_1^2}{3} \qquad \left(\because\ i_1 = \int_0^{r_1} di = \int_0^{r_1} (br)(2\pi r\, dr) = \frac{2\pi b r_1^3}{3}\right)$$

29. 32

Sol. $\text{Pitch} = \frac{2\pi m v \cos\theta}{qB} \qquad eV = \frac{1}{2} m v^2$

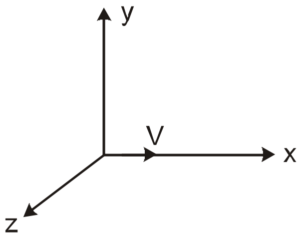
$$= \frac{2\pi\sqrt{2meV}}{eB} \qquad (\cos\theta \approx 1)$$

$$\text{Pitch} = \sqrt{\frac{8\pi^2 mV}{eB^2}}$$

$\therefore$ Distance from point of divergence $= \sqrt{\frac{32\pi^2 mV}{eB^2}}$

30. 10

Sol. Magnetic force is in $\hat{j}$ direction and electric field is in $-\hat{j}$ direction



Resultant force $= qVB - qE$

$= q(1.28 \times 10^{6} \times 8 \times 10^{-2} - 102.4 \times 10^{3})$

$= 0$

$R = 100\,m$

Charge will move only in x direction.

$x = V \times t = 1.28 \times 10^{6} \times 5 \times 10^{-6} = 6.4\,m$

Now electric fields is switched off.

31. (B)

Sol. $aA + bB \longrightarrow \text{Product}$

$$\frac{dx}{dt} = k[A]^a[B]^b$$

(i) As on doubling concentration of A, rate become four time so a = 2.

(ii) On increasing the concentration of B four times, rate become double so $b = \frac{1}{2}$

So, Given equation :

$$2a + \frac{1}{2}b \longrightarrow \text{Product}$$

$$-\frac{1}{2}\frac{d[A]}{dt} = -2\frac{d[B]}{dt}$$

$$-\frac{d[A]}{dt} = -4\frac{d[B]}{dt}$$

32. (C)

Sol. As $t_{50\%}$ is constant, hence order of reaction is 1.

$$n = 1,\ t_{1/2} = \frac{0.693}{k}$$

33. (B)

Sol. Let $r = (A)^x (B)^y$

$$x = \frac{\log\left(\frac{r_1}{r_2}\right)}{\log\left(\frac{a_1}{a_2}\right)} = \frac{\log\frac{0.1}{0.1}}{\log\left(\frac{0.012}{0.024}\right)} = \frac{\log\left(\frac{1}{8}\right)}{\log\left(\frac{1}{2}\right)}$$

$x = 3$

$$y = \frac{\log\frac{r_1}{r_2}}{\log\left(\frac{b_1}{b_2}\right)} = \frac{\log\left(\frac{0.1}{0.1}\right)}{\log\left(\frac{0.035}{0.070}\right)} = \frac{\log(1)}{\log\left(\frac{1}{2}\right)}$$

$y = 0$

34. (B)

Sol. $NH_4NO_2(aq) \longrightarrow N_2(g) + 2H_2O(\ell)$

$$k = \frac{2.303}{t}\log\left(\frac{V_\infty - V_o}{V_\infty - V_t}\right) \qquad \Rightarrow V_o = 0$$

$$k = \frac{2.303}{t}\log\left(\frac{70}{30}\right) \qquad k = \frac{2.303}{t}\log\frac{7}{3}$$

35. (B)

Sol. In given sequence of reaction

$$\frac{d[C]}{dt} = k_2[B] - k_3\ [C]$$

36. (B)

Sol.
$$\% \text{of B} = \frac{k_1 \times 100}{k_1 + k_2} \frac{1.26 \times 100}{12.6 \times 10^{-5} + 3 \times 10^{-5}} 76.83\%$$

$$\% \text{of C} = \frac{k_2 \times 100}{k_1 + k_2} \frac{3 \times 10^{-5} \times 100}{12.6 \times 10^{-5} + 3 \times 10^{-5}} 23.18\%$$

37. (A)

Sol. Zn is a weakest oxidizing agent according to Electrochemical series. So its a strongest reducing Agent.

38. (B)

Sol. Increase in the concentration of Ag^+ ion.

39. (D)

Sol.
$$A : Zn \longrightarrow Zn^{+2} + 2e^-$$
$$C : 2Ag^+ + 2e^- \longrightarrow 2Ag$$
$$Zn + 2Ag^+ \longrightarrow 2Ag + Zn^{+2}$$

$$Q_C = \frac{[Ag]^2[Zn^{+2}]}{[Ag^+]^2[Zn]} = \frac{10^{-2}}{(1.25)^2} = 6.40 \times 10^{-3}$$

40. (B)

Sol. $E°_{OP}$ of Zn > $E°_{OP}$ of Fe

41. (C)

Sol.
$$\frac{1.81 \times 10^{22}}{6.02 \times 10^{23}} = \frac{1}{\text{At. mass}} \times 2$$

reaction $Mn^{+2} \rightarrow Mn$

$n_f = 2$

$$\text{At. mass} = \frac{2 \times 6.023 \times 10^{23}}{1.81 \times 10^{22}} = 66.7$$

42. (B)

Sol. Volume : $10 \times 10 \times 10^{-2} = 1\ cm^3$

mass of Cu = 8.94 g

$$\text{mole of Cu} = \frac{8.94}{63.5}$$

$$\text{Charge} = \frac{8.94}{63.5} \times 2 \times 96500$$

=27172 C

43. (C)

Sol. At Cathode

$H_2O + 4e^- \rightarrow 4H_2 + 4OH^-$

At anode

$2H_2O \rightarrow 4H^+ + O_2 + 4e^-$

Thus at cathode we will get H_2

whereas at

anode we will get O_2.

44. (C)

Sol. I is for strong electrolyte and II for weak electrolyte.

45. (A)

Sol. Let us suppose K_1 & K_2 are speci fic conductance of solution A & B respectively.

Cell constant is 'y'

We know that

specific conductance = conductance × cell constant

For (A) , $K_1 = 1/40 \times y$

For (B), $K_2 = 1/70 \times y$

When equal volume of solution A & B are mixed then volume becomes double then

$$\text{specific conductance of mixture} = \frac{K_1 + K_2}{2}$$

$$\therefore \frac{K_1 + K_2}{2} = \frac{1}{R} \times y \ \& \ \frac{1}{2}\left[\frac{y}{40} + \frac{y}{70}\right] = \frac{1}{R} y$$

$$\therefore \frac{1}{2}\left[\frac{11y}{280}\right] = \frac{y}{R}$$

$$R = \frac{280 \times 2}{11} = 50.9\Omega$$

46. (A)

Sol. By Kohlrausch's law

$\lambda° NaBr = \lambda° NaCl + \lambda° KBr - \lambda° KCl$

$= 126 + 152 - 150$

$= 128 \text{ S cm}^2 \text{ mol}^{-1}$

47. (C)

Sol. Cell is completely discharged, so $E^{\circ}_{cell} = 0$

$Zn|Zn^{2+}(1M)||Cu^{2+}(1M)|Cu$

Cell reaction

$Zn + Cu^{2+} \rightleftharpoons Zn^{2+} + Cu$

$$K_{eq} = \frac{[Zn^{2+}]}{[Cu^{2+}]}$$

We know

$$E_{cell} = E^{\circ}_{cell} - \frac{0.0591}{n} \log K_{eq}$$

$$1.10 = E^{\circ}_{cell} - \frac{0.0591}{2} \log k$$

$$K_{eq} = \frac{[Zn^{2+}]}{[Cu^{2+}]} = \text{anti log } \frac{2.20}{0.0591}$$

$\therefore$ anti log 37.3

$\Delta K_{eq} = 10^{37.3}$

48. (A)

Sol. According to Debye-Huckel theory, for strong electrolyte like NaCl, $\lambda_C = \lambda_\infty \sqrt{C}$

49. (D)

Sol. $B_2H_6 + 3O_2 \rightarrow B_2O3 + 3H_2O$

1 mol 3 mol

3mol O_2 is required for burring 1 mol B_2H_6

$H_2O \xrightarrow{\text{Electrolysis}} H_2 + \frac{1}{2}O_2$

$(n_f \text{ of } O_2 = 4)$

$\frac{\text{Equivalent of } O_2}{n_f \text{ of } O_2} = \text{mol of } O_2 = 3$

$\left[\frac{100 \times t_{sec}}{96500}\right] \times \frac{1}{4} = 3$

$\therefore \; t = \frac{3 \times 96500}{100 \times 3600} \text{hr} = 3.2 \text{ hr.}$

50. (D)

Sol. $\frac{d}{dt}[SO_3] 100g/min = \frac{100}{80} mol/min = 1.25 mol/min$

$\frac{-1}{2}\frac{d}{dt}[SO_2] = \frac{-d}{dt}[O_2] = \frac{1}{2}\frac{d}{dt}[SO_3]$

51. 2

Sol. $E_{cell} = E^\circ_{cell} - \frac{0.059}{2}\log\frac{[M^{+2}][H^+]^2}{[M^{+4}][P_{H_2}]}$

$0.092 = 0.151 - \frac{0.059}{2}\log 10^x$

$X = 2$

52. 2

Sol. $2A + B \longrightarrow C$

Unit of k = L mol^{-1} s^{-1}

So order of reaction = 2.

53. 8

Sol. $A_2(g) \rightarrow B(g) + \frac{1}{2}C(g)$

t = 0   100mm   0   0

t = 5 min   (100 – x)   x $\frac{x}{2}$

Total pressure= $100 - x + x + \frac{x}{2} = 120$

(x = 40)

then Rate of disappearance of A_2

$= -\frac{d[A_2]}{dt} = \frac{40}{5} = 8$

54. 3

Sol. Since to deposit 1 mole of aluminium 3 coulomb of electricity is required, as the valency of silver is +1 so 3 mole of silver will be deposited by 3 coulomb of electricity.

55. 391

Sol. $\lambda^{\infty}_{CH_3COOH} = \lambda^{\infty}_{HCl} + \lambda^{\infty}_{CH_3COONa} - \lambda^{\infty}_{NaCl}$

$= 426 + 91 - 126$

$= 517 - 126 = 391\ \Omega^{-1}\ cm^2 mol^{-1}$

56. 0.29

Sol. $t_{1/4} = \frac{2.303}{k} \log \frac{1}{1-\frac{1}{4}} = 0.29 / k$

57. 2

Sol. $NO(g) + Br_2(g) \rightleftharpoons NOBr_2\ (g)$

$NOBr_2(g) + NO(g) \longrightarrow 2NOBr(g)$

Rate of reaction(r) = k[$NOBr_2$] [NO]

where [NOBr2] = k_C [NO] [Br2]

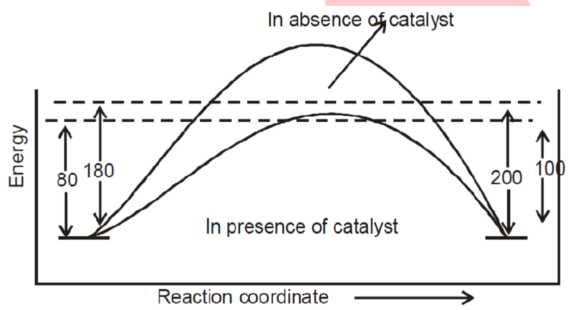
r = k. k_C [NO] [Br_2] [NO]

r = k' $[NO]^2$ [Br_2]

The order of the reaction with respect to NO = 2

58. 20

Sol.



So ΔH reaction $= E_F - E_B = 80 - 100 = -20$

59. 0.25

Sol. For a zero order rate constant $k = \frac{X}{t}$ ....(1)

Where x = amount decomposed

k = zero order rate constant

for a zero order Reaction

$k = \frac{[A]}{2 \times t_{\frac{1}{2}}}$.....(2)

Since $[A_0] = 2M$, $t_{\frac{1}{2}} = 1$ h

k = 1

$\therefore$ From equation (1)

$t = \frac{0.25}{1} = 0.25$ h

60. 4

Sol. $k_2 = Ae^{\frac{E_{a_2}}{RT}} \Rightarrow k_1 = Ae^{\frac{E_{a_1}}{RT}} \Rightarrow \frac{k_2}{k_1} = e^{\frac{E_{a_1} - E_{a_2}}{RT}}$

$\Rightarrow \log \frac{k_2}{k_1} = \frac{E_{a_1} - E_{a_2}}{RT} = \frac{10 \times 1000}{8.3 \times 300} \simeq \frac{100}{25} \simeq 4$

61. (B)

Sol. $$\lim_{x\to\infty}\frac{x^2\sin\left(\log_e\sqrt{\cos\frac{\pi}{x}}\right)}{\log_e\sqrt{\cos\frac{\pi}{x}}}\cdot\log_e\sqrt{\cos\frac{\pi}{x}}$$

$$=\lim_{x\to\infty}\frac{x^2}{2}\frac{\log_e\left[1+\left(\cos\frac{\pi}{x}-1\right)\right]}{\left(\cos\frac{\pi}{x}-1\right)}\left(-2\sin^2\frac{\pi}{2x}\right)=-$$

62. (C)

Sol. This problem requires a geometrical argument :

Method.1 By similar triangles, $\frac{f(x)}{6}=\frac{x-a}{0-a}=\frac{g(x)}{3}$, and therefore $\frac{f(x)}{g(x)}=\frac{6}{3}=2$

Method.2 $$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{\frac{f(x)}{-a}}{\frac{g(x)}{-a}}=\lim_{x\to a}\frac{\text{slope of } f}{\text{slope of } g}=\frac{6}{3}=2$$

63. (C)

Sol. $$\lim_{x\to 0}\left\{\frac{f(1+x)}{f(1)}\right\}^{1/x}$$

$$=\lim_{x\to 0}\left\{1+\frac{f(1+x)-f(1)}{f(1)}\right\}^{1/x}$$

$$=e^{\lim_{x\to 0}\frac{f(x+1)-f(1)}{x\,f(1)}}$$

$$=e^{\frac{1}{f(1)}\lim_{x\to 0}\frac{f(x+1)-f(1)}{(x)}}$$

$$=e^{\frac{f'(1)}{f(1)}}\left[\because \lim_{x\to 0}\frac{f(1+x)-f(1)}{x}=f'(1)\right]$$

$$=e^{6/3}=e^2$$

64. (A)

Sol. $$\lim_{x\to 0}\frac{ae^{2x}-b\cos 2x+ce^{-2x}-x\sin x}{x\sin x}=1$$

$$\lim_{x\to 0}\frac{\left(1+2x+\frac{(2x)^2}{2!}+.....\right)-b\left(1+\frac{(2x)^2}{2!}+\frac{(2x)^4}{4!}-.....\right)+c\left(1-2x+\frac{(2x)^2}{2!}-.....\right)}{x^2}=2$$

$\Rightarrow a-b+c=0$ ....(1)

$2a-2c=0$ ....(2)

$2a+2b+2c=2$ ....(3)

$\Rightarrow a=c$ and $b=2a$

$\Rightarrow a+2a+a=1 \quad\Rightarrow\quad a=1/4=c$

$b=\frac{1}{2}$

$f(t)=\frac{3}{4}t^2-\frac{1}{4}t+\frac{1}{4}=\frac{3t^2-t+1}{4}$

65. (A)

Sol. Using L-Hospital's rule, we get

$$-1=\lim_{x\to a}\frac{a^x-x^a}{x^x-a^a}=\lim_{x\to a}\frac{a^x\log_e a-ax^{a-1}}{x^x+a^a\log_e a}$$

$$\Rightarrow -1=\frac{a^a\log_e a-a.a^{a-1}}{a^a+a^a\log_e a}=\frac{\log_e a-1}{\log_e a+1} \quad .....(i)$$

Now (i) is satisfied only when $a=1$.

66. (B)

Sol. **Aliter :** Apply L-Hospital's rule,

67. (C)

Sol. $f'(x)=2x[a_1+2a_2x^2+.....+na_nx^{2n-2}]=0$

$\Rightarrow$ $x=0$ which is the point of local minima

68. (D)

Sol. $\ell im_{x\to 5}\frac{(f(x)^2)-9}{\sqrt{|x-5|}}=0$; $\ell im_{x\to 5}[(f(x))^2-9]=0$; $\ell im_{x\to 5} f(x)=3$

69. (A)

Sol. $\ell n\, f(x)>0\Rightarrow f(x)>1$

$f'(x)+21\, f(x)>0 \quad x\in R$

$\Rightarrow e^{21x}f'(x)+21\, e^{21x}f(x)>0 \;\forall\; x\in R$

$\Rightarrow \frac{d}{dx}(f(x)\, e^{21x})>0 \;\forall\; x\in R$

$\Rightarrow$ g(x) is an increasing function

70. (C)

Sol. $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$

$$\Rightarrow \lim_{x\to 0}\frac{x\left\{1+a\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+...\right)\right\}-b\left\{x-\frac{x^3}{3!}+\frac{x^5}{5!}-...\right\}}{x^3}=1$$

$$\Rightarrow \lim_{x\to 0}\frac{(1+a-b)+x^2\left(\frac{b}{3!}-\frac{a}{2!}\right)+x^4\left(\frac{a}{4!}-\frac{b}{5!}\right)+...}{x^2}=1 \quad .....(i)$$

If $1+a-b\neq 0$, then L.H.S. $\to\infty$ as $x\to 0$ while R.H.S. =1, therefore $1+a-b=0$.

Now from (i), $\lim_{x\to 0}\frac{x^2\left(\frac{b}{3!}-\frac{a}{2!}\right)+x^4\left(\frac{a}{4!}-\frac{b}{5!}\right)+...}{x^2}=1$

$\Rightarrow \frac{b}{3!}-\frac{a}{2!}=1\Rightarrow b-3a=6$. Solving $1+a-b=0$ and $b-3a=6$, we get $a=-5/2, b=-3/2$.

71. (A)

Sol. Doubtful points are $x=n,\ n\in I$

$$\text{L.H.L}=\lim_{x\to n^-}[x]\cos\left(\frac{2x-1}{2}\right)\pi=(n-1)\cos\left(\frac{2n-1}{2}\right)\pi=0$$

$$\text{R.H.L.}=\lim_{x\to n^+}[x]\cos\left(\frac{2n-1}{2}\right)\pi=n\cos\left(\frac{2n-1}{2}\right)\pi=0$$

$f(n)=0$

Hence continuous

72. (C)

Sol. $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^n}{n!}$

$\Rightarrow \frac{dy}{dx} = 0 + 1 + x + \frac{x^2}{2!} + ... + \frac{x^{n-1}}{(n-1)!}$

$\Rightarrow \frac{dy}{dx} + \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + ... + \frac{x^n}{n!} \Rightarrow \frac{dy}{dx} = y - \frac{x^n}{n!}$.

73. (D)

Sol. $y = a\sin x + b\cos x$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = a\cos x - b\sin x$$

Now $\left(\frac{dy}{dx}\right)^2 = (a\cos x - b\sin x)^2$

$$= a^2\cos^2 x + b^2\sin^2 x - 2ab\sin x\cos x$$

and $y^2 = (a\sin x + b\cos x)^2$

$$= a^2\sin^2 x + b^2\cos^2 x + 2ab\sin x\cos x$$

So, $\left(\frac{dy}{dx}\right)^2 + y^2 = a^2(\sin^2 x + \cos^2 x) + b^2(\sin^2 x + \cos^2 x)$

Hence $\left(\frac{dy}{dx}\right)^2 + y^2 = (a^2 + b^2) =$ constant.

74. (B)

Sol. $f(0) = 0$ and $f(x) = xe^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}$

R.H.L. $= \lim_{h \to 0}(0 + h)e^{-2/h} = \lim_{h \to 0}\frac{h}{e^{2/h}} = 0$

L.H.L. $= \lim_{h \to 0}(0 - h)e^{-\left(\frac{1}{h} - \frac{1}{h}\right)} = 0$; $\therefore f(x)$ is continuous.

$$Rf'(x) = \lim_{h \to 0}\frac{(0 + h)e^{-\left(\frac{1}{h} + \frac{1}{h}\right)} - he^{-\left(\frac{1}{h} + \frac{1}{h}\right)}}{h} = 0$$

$$Lf'(x) = \lim_{h \to 0}\frac{(0 - h)e^{-\left(\frac{1}{h} - \frac{1}{h}\right)} - he^{-\left(\frac{1}{h} + \frac{1}{h}\right)}}{-h} = 1$$

$\Rightarrow Lf'(x) \neq Rf'(x)$. is not differentiable at

75. (A)

Sol. $y = \frac{e^{2x}\cos x}{x\sin x} \Rightarrow \log y = 2x + \log\cos x - \log x - \log\sin x$

$\frac{1}{y}\frac{dy}{dx} = 2 + \left(\frac{-\sin x}{\cos x}\right) - \frac{1}{x} - \frac{\cos x}{\sin x}$

$\Rightarrow \frac{dy}{dx} = e^{2x}\left[\frac{2}{x}\cot x - \frac{1}{x} - \frac{1}{x^2}\cot x - \frac{\cot^2 x}{x}\right]$

$= \frac{e^{2x}}{x^2}[(2x - 1)\cot x - x\,\mathrm{cosec}^2 x]$.

76. (C)

Sol. We are given that $xe^{xy} = y + \sin^2 x$

When $x = 0$, we get $y = 0$

Differentiating both sides with respect to *x*, we get

$$e^{xy} + xe^{xy}\left[x\frac{dy}{dx} + y\right] = \frac{dy}{dx} + 2\sin x \cos x$$

Putting $x = 0, y = 0$, we get $\frac{dy}{dx} = 1$.

77. (B)

Sol. $y = x^{x^{x.......\infty}} \Rightarrow y = x^y \Rightarrow \log y = y \log x$

Therefore, on differentiating $\frac{dy}{dx} = \frac{y^2}{x(1 - y\log x)}$.

78. (D)

Sol. $I_n = \frac{d^{n-1}}{dx^{n-1}}[x^{n-1} + nx^{n-1}\log x]$

$I_n = (n-1)! + nI_{n-1} \Rightarrow I_n - nI_{n-1} = (n-1)!$.

79. (C)

Sol. Here $V = \frac{4}{3}\pi r^3$ and $S = 4\pi r^2$

$\Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{40}{4\pi r^2} = \frac{5}{32\pi}$

$\therefore \frac{dS}{dt} = 8\pi r \frac{dr}{dt} = 8\pi \times 8 \times \frac{5}{32\pi} = 10$.

80. (C)

Sol. $f(x) = [x]\cos\left[\frac{2x-1}{2}\right]\pi$

Since $g(x) = [x]$ is always discontinuous at all integral values of points. Hence $f(x)$ is discontinuous for all integral points.

81. 5

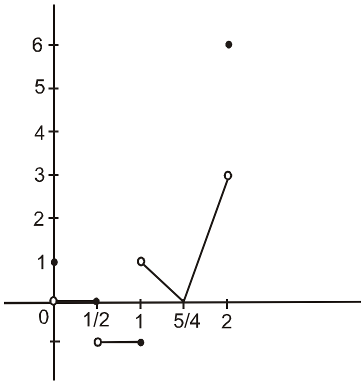
Sol. $\frac{dx}{dy} = \frac{\frac{dx}{dt}}{\frac{dy}{dt}} = \frac{12t^2}{12t^3} = \frac{1}{t}$

$$\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dt}\left(\frac{1}{t}\right).\frac{dt}{dy} = \frac{-1}{t^2}.\frac{1}{12t^3} = \frac{-1}{12t^5}$$

So $\frac{\frac{-1}{12t^5}}{\left(\frac{1}{t}\right)^n}$ is constant $\Rightarrow n = 5$

82. 4

Sol. $f(x)=\begin{cases} 1 & , \quad x=0 \\ 0 & , \quad 0<x\le 1/2 \\ -1 & , \quad 1/2<x\le 1 \\ 5-4x & , \quad 1<x<5/4 \\ 4x-5 & , \quad 5/4\le x<2 \\ 6 & , \quad x=2 \end{cases}$



f(x) is discontinous at x = 0, 1/2, 1, 2 in [0, 2]

83. 2

Sol. $\lim\limits_{n\to\infty}\left(\frac{1}{\sqrt{n^2}}+\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+............+\frac{1}{\sqrt{n^2+2n}}\right)$

using sandwitch theorem

$\frac{1}{\sqrt{n^2}}\le\frac{1}{n}$

$\frac{1}{\sqrt{n^2+1}}\le\frac{1}{n}$

$\vdots \qquad \vdots$

$\frac{1}{\sqrt{n^2+2n}}\le\frac{1}{n}$

adding all these inequilities

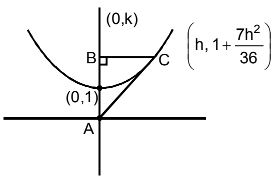
$\frac{1}{\sqrt{n^2}}+\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+............+\frac{1}{\sqrt{n^2+2n}}\le\frac{2n}{n}$

Taking both side $\lim\limits_{n\to\infty}$

$\lim\limits_{n\to\infty}\left(\frac{1}{\sqrt{n^2}}+\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+............+\frac{1}{\sqrt{n^2+2n}}\right)=2$

84. 66

Sol. at t = 0 ; x = 0, y = 1



$\frac{dy}{dt}=2\text{cm/sec}$

$$A=\frac{1}{2}\times h\times\left(1+\frac{7h^2}{36}\right)$$

$$\frac{dA}{dt}=\left(\frac{1}{2}\left(1+\frac{7h^2}{36}\right)+\frac{h}{2}\left(\frac{14h}{36}\right)\right)\frac{dh}{dt} \quad ; \quad \frac{dA}{dt}=\left(\frac{1}{2}\times 8+3\times\frac{14\times 6}{36}\right)\times\frac{6}{(7/2)}$$

(At, t = 7/2 sec, change in y -co-ordinate = 7 hence, pt. C has

y-co-ordinate = 8 and x- co-ordinate = 6 at t = 7/2 sec.) $=(4+7)\times\frac{6}{7}\times 2=\frac{132}{7}$ cm²/sec

85. 36

Sol. $f\left(\frac{\pi}{2}\right)=\lim_{h\to 0}\frac{1-\cosh}{4h^2}\frac{\ell n\ (\cosh)}{\ell n\ [1+4h^2]}$

$$=\lim_{h\to 0}\frac{2}{16\times 16}\left(\frac{\sin^2 h/2}{h^2/2}\right).\frac{4h^2}{\ell n\ (1+4h^2)}.\frac{\ell n(1-2\sin^2 h/2)}{2\sin^2 h/2}.\frac{2\sin^2 h/2}{h^2/2}$$

$$=\frac{1}{64}\ .1.1.(-1).1=-\frac{1}{64}$$

$\Rightarrow \quad \alpha^{\beta}=64 \quad =2^6, 4^3, 8^2, 64^1$

86. 1

Sol. $\lim_{x\to\infty}\left(x\sin\left(\frac{1}{x}\right)+\sin\left(\frac{1}{x^2}\right)\right)$

$$\lim_{x\to\infty}\left(\frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}}+\sin\left(\frac{1}{x^2}\right)\right)=1+0=1$$

87. 7

Sol. $\lim_{x\to 1^-}f(x)=\lim_{x\to 1}\ x^2e^{2(x-1)}=1$

$f(1)=1$

$\lim_{x\to 1^+}f(x)=\lim_{x\to 1}$ a sgn (x +1) cos2(x–1) + bx² = a.1.1+ b

for continuity a + b = 1

LHD (x = 1) is $\lim_{h\to 0}\frac{(1-h)^2e^{-2h}-1}{h}=\lim_{h\to 0}2e^{-2h}+he^{-2h}+\left(\frac{e^{-2h}-1}{h}\right)$

$$=2+0+2=4$$

RHD (x = 1) is $\lim_{h\to 0}\frac{a\,sgn(2+h)\cos 2h+b(1+h)^2-1}{h}=\lim_{h\to 0}\frac{a\cos 2h+b+bh^2+2bh-(a+b)}{h}$

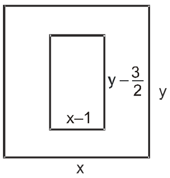
$$=\lim_{h\to 0}a\left(\frac{\cos 2h-1}{h}\right)+bh+2b=2b$$

f(x) is differentiable at x = 1 if 2b = 4 $\qquad$ b = 2 $\qquad$ a = –1

88. 39

Sol. $xy = 18$

Area of printed space $= (x-1)\left(y-\frac{3}{2}\right) \quad = 18+\frac{3}{2}-\left(\frac{3x}{2}+\frac{18}{x}\right)$



Figure

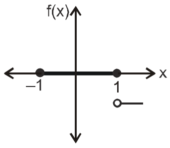
Maximum when $\frac{3x}{2}=\frac{18}{x}$

$\Rightarrow \quad x = 2\sqrt{3} \qquad y = 3\sqrt{3}$

89. 3

Sol. $f(x) = [x \sin \pi x]$

graph of f(x) is as shown in the figure



90. 481

Sol. $f(x)=\sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$ at $x = 1$; $22x = 4$

for $\sin\left(\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right)$

Let $\tan^{-1}x = \theta, \theta \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\sin(\cos^{-1}\cos2\theta) = \sin2\theta$

$$\left\{\begin{array}{ll} \text{if} & x>1 \Rightarrow \frac{\pi}{2}>\theta>\frac{\pi}{4} \\ \therefore & \pi>2\theta>\frac{\pi}{2} \end{array}\right\}$$

$= 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

$= \frac{2x}{1+x^2}$

Hence, $f(x)=\frac{2.2^x}{1+2^{2x}}$

$$\therefore f'(x)=\frac{(1+2^{2x})(2.2^x\ln2)-2^{2x}\cdot2\cdot\ln2\cdot2\cdot2^x}{(1+2^{2x})}$$

$$\therefore f^1(1)=\frac{20\ln2-32\ln2}{25}=-\frac{12}{25}\ln2$$

So, $a = 25, b = 12 \Rightarrow |a2 - b2| = 252 - 122$

$= 625 - 144$

$= 481$