

JEE MAIN ANSWER KEY & SOLUTION**PAPER CODE :- PART TEST-3
CLASS-XII****ANSWERKEY****PHYSICS**

1.	(C)	2.	(C)	3.	(B)	4.	(A)	5.	(B)	6.	(A)	7.	(D)
8.	(B)	9.	(B)	10.	(C)	11.	(D)	12.	(D)	13.	(A)	14.	(D)
15.	(D)	16.	(A)	17.	(A)	18.	(D)	19.	(C)	20.	(B)	21.	4
22.	4	23.	0	24.	12	25.	6	26.	3	27.	8	28.	4
29.	2	30.	1										

CHEMISTRY

31.	(D)	32.	(C)	33.	(C)	34.	(A)	35.	(A)	36.	(D)	37.	(C)
38.	(B)	39.	(C)	40.	(A)	41.	(B)	42.	(B)	43.	(D)	44.	(A)
45.	(B)	46.	(C)	47.	(C)	48.	(B)	49.	(A)	50.	(B)	51.	95
52.	7	53.	4	54.	3	55.	5	56.	5	57.	35	58.	2
59.	15	60.	18										

MATHEMATICS

61.	(A)	62.	(C)	63.	(B)	64.	(B)	65.	(B)	66.	(C)	67.	(A)
68.	(D)	69.	(C)	70.	(A)	71.	(C)	72.	(C)	73.	(B)	74.	(A)
75.	(B)	76.	(B)	77.	(C)	78.	(A)	79.	(A)	80.	(B)	81.	5
82.	16	83.	12	84.	158	85.	10	86.	7	87.	2	88.	2
89.	16	90.	3										

PE

SOLUTIONS

PHYSICS

1. (C)

Sol. Cathode rays

2. (C)

Sol.
$$\varepsilon = \frac{B\omega R^2}{2}$$

$$= \frac{0.1 \times (2\pi \times 20) (0.1)^2}{2}$$

$$= 20\pi \times 10^{-3} \text{ volt}$$

3. (B)

Sol. $\lambda_x < \lambda_v < \lambda_m$

4. (A)

Sol.
$$U = \frac{1}{2} LI_2$$

$$\text{Rate} = \frac{dU}{dt} = LI \left(\frac{dI}{dt} \right)$$

At $t = 0$, $I = 0$

\therefore Rate = 0

At $t = \infty$, $I = I_0$ but $\frac{dI}{dt} = 0$, therefore, rate = 0

5. (B)

Sol.
$$T = \frac{2\pi r}{v}$$

$$\left. \begin{array}{l} r \propto n^2 \\ v \propto 1/n \end{array} \right| v \propto \frac{1}{\sqrt{r}} \Rightarrow T \propto \frac{r}{v} \propto r^{3/2}$$

6. (A)

Sol. QV because induced electric field so generated is non conservative i.e. $\oint E \cdot dl = V$.

7. (D)

8. (B)

Sol. $\phi_1 = BA \cos 0^\circ = BA$

$\phi_2 = B(0) \cos \theta = 0$

$$\Rightarrow q_{\text{ind}} = - \frac{d\phi}{R} = \frac{BA}{R}$$

9. (B)

10. (C)

Sol.
$$E = L \frac{di}{dt}$$

$$di = \frac{E}{L} dt$$

$$i = \frac{E}{L} t$$

$$i = \frac{2}{4} \times t$$

$$i = 0.5t$$

$$5 = 0.5t$$

$$t = 10 \text{ sec}$$

11. (D)
 Sol. Time taken by laser beam to move through a distance 90cm is

$$t = \frac{90}{c} = \frac{90\text{cm}}{3 \times 10^{10} \text{cm/s}} = 3 \times 10^{-9} \text{s}$$

The energy contained in 90cm length of laser beam is
 $U = Pt = (10\text{mW}) \times (3 \times 10^{-9}\text{s})$
 $= (10 \times 10^{-3} \text{Js}^{-1}) \times (3 \times 10^{-9}\text{s})$
 $= 30 \times 10^{-12} \text{J}$

12. (D)

Sol. Since, $\cos\theta = \frac{R}{Z} = \frac{IR}{IZ} = \frac{8}{10} = \frac{4}{5}$

($\cos\theta$ can never be greater than 1)

Also, $I_{X_C} > I_{X_L} \Rightarrow X_C > X_L$
 Current will be leading

In a LCR circuit

$$V = \sqrt{(V_L - V_C)^2} = \sqrt{(6 - 12)^2 + 8^2}$$

$V = 10$; which is less than voltage drop across capacitor .

13. (A)

Sol. $\epsilon_1 = \frac{\mu_0 I a v}{2\pi x}$

and $\epsilon_2 = \frac{\mu_0 I a v}{2\pi(x+a)}$

$$\epsilon_{\text{net}} = \epsilon_1 - \epsilon_2$$

$$= \frac{\mu_0 I a v}{2\pi} \left[\frac{1}{x} - \frac{1}{x+a} \right]$$

$$= \frac{\mu_0 I a^2 v}{2\pi x(x+a)}$$

PE

14. (D)

Sol. $i_0 = \frac{V_0}{Z}$,

$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$= \sqrt{4^2 + (1000 \times 3 \times 10^{-3})^2} = 5 \Omega$$

$$i_0 = \frac{4}{5}$$

$$i_0 = 0.8 \text{A}$$

15. (D)

16. (A)

Sol. From Kirchoff's current law,

$$i_3 = i_1 + i_2 = 3 \sin \omega t + 4 \sin (\omega t + 90^\circ)$$

$$= \sqrt{3^2 + 4^2 + 2(3)(4) \cos 90^\circ} \sin(\omega t + \phi)$$

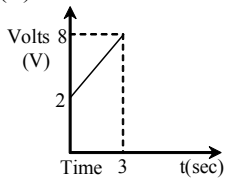
$$\text{where } \tan \phi = \frac{4 \sin 90^\circ}{3 + 4 \cos 90^\circ} = \frac{4}{3}$$

$$\therefore i_3 = 5 \sin(\omega t + 53^\circ)$$

17. (A)

18. (D)

Sol.



$$I = \frac{dq}{dt}$$

$$q = it + a$$

$$V = \frac{q}{c}$$

$$V = \frac{it + a}{c}$$

∴ V is proportional to time.

19. (C)

20. (B)

Sol.

$$P = V_{\text{rms}} I_{\text{rms}} \cos\phi$$

$$P = V_{\text{rms}} \frac{V_{\text{rms}}}{Z} \frac{R}{Z}$$

$$= \frac{V_{\text{rms}}^2}{Z^2} R$$

$$Z = \sqrt{R^2 + (L\omega)^2} = \sqrt{(50)^2 + (0.2 \times 250)^2}$$

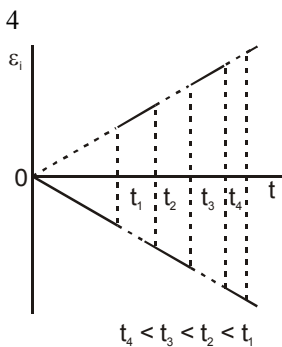
$$= \sqrt{2500 + (50)^2} = 50\sqrt{2}$$

$$\therefore P = \left(\frac{200}{\sqrt{2}}\right)^2 \times \frac{50}{50\sqrt{2}} \times \frac{1}{50\sqrt{2}}$$

$$= 200 \text{ watt}$$

21.

Sol.



$$\text{Flux} = \phi = \vec{A} \cdot \vec{B} = -\left(\frac{a^2}{2}\theta\right) B$$

$$\phi = -\frac{a^2}{2} \left(\frac{\beta t^2}{2}\right) B$$

$$e = -\frac{d\phi}{dt} = \frac{a^2 B \beta t}{2} (-1)^n$$

Put values

$$x=4$$

22. 4

Sol.
$$EMF = -\frac{d\phi}{dt} = \left(\frac{\pi a^2 \mu_0 \omega}{2b}\right)^2 \frac{l \cos 2\omega t}{R}$$

23. 0

Sol. $z = x + y i + \omega L i$
 $= x + i(y + \omega L)$ for power factor to be one $y + \omega L = 0 \Rightarrow y = -10$

$$I = \frac{V_0}{x}, \quad x = \frac{V_0}{I} = \frac{25}{5} = 5$$

Impedance of box = $5 - 10 i$

$$\cos\phi = \frac{5}{\sqrt{10^2 + 5^2}} = \frac{1}{\sqrt{5}} = 0.447$$

24. 12

Sol. $EMF = -\frac{d\phi}{dt} = (B_2 - B_1) \ell v$

$$EMF = \ell v \left(\frac{\mu_0 i}{2\pi a} - \frac{\mu_0 i}{2\pi(a + \ell)} \right) = \frac{\mu_0 i^2 v}{2\pi a(a + \ell)}$$

Put values $x = 12$

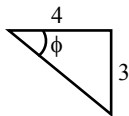
25. 6

Sol. $Z = R - j \times C$
 $Z = 4 - 3j$

$$Z = 5 \angle \tan^{-1}\left(\frac{-3}{4}\right) = 5 \angle -37^\circ$$

$$I = \frac{V_0 \sin \omega t}{Z}$$

$$I = \frac{V_0}{Z} \sin(\omega t + \phi) \text{ where } \tan \phi = \frac{3}{4} \Rightarrow \phi = 37^\circ$$



phase difference between I and $I_1 = 37^\circ$

26. 3

Sol. $\int (\text{Torque of elementary force}) = \text{Torque of net force.}$

$$\frac{\mu_0 I_0}{2\pi} I \int \frac{x dx}{x} = F_0 \bar{x} = \frac{\mu_0 I_0}{2\pi} I \ln \left(\frac{b}{a} \right) \bar{x}$$

$$\Rightarrow \bar{x} = \frac{b - a}{\ln(b/a)}$$

$$\Rightarrow \bar{x} = \frac{2a}{\ln 3}$$

$$x = 3$$

27. 8

Sol. At resonance reactance = 0

$$I = \frac{V}{R} = \frac{60}{120} = \frac{1}{2} \text{ Amp.}$$

$$V_L = I \times X_L = I \times \omega L$$

$$\therefore L = \frac{V_L}{I\omega} \quad \dots\dots\dots(i)$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$C = \frac{1}{L\omega_0^2} \quad \dots\dots\dots(ii)$$

Calculate L & C from (1) & (2) current will lag the applied voltage by 45°

$$\text{if } \tan 45^\circ = \frac{\omega L - \frac{1}{\omega C}}{R}$$

Solve for ω $\omega = 8 \times 10^5 \text{ rad/sec}$

28. 4

Sol.

$$\tau = M \times B$$

$$= MB \sin 90^\circ$$

$$= Ni\pi R^2 B$$

$$2\pi RN = \ell$$

$$R = \frac{\ell}{2\pi N}$$

$$\tau = Ni\pi \frac{\ell^2}{2\pi^2 N^2} B$$

$$\tau_{\max} \Rightarrow N = 1$$

$$\Rightarrow t = B$$

$$[N = 4]$$



29. 2

Sol.



$$I_{\text{Net}} = I_L - I_C \\ = 0.8 - 0.6 \\ = 0.2 \text{ Amp.}$$

30. 1

Sol.

Instantaneous flux

$$= \pi a^2 B \cos 0^\circ + \pi b^2 B \cos 180^\circ$$

$$= \pi (a^2 - b^2) B$$

$$\phi = \pi (a^2 - b^2) B_0 \sin \omega t$$

$$\ell = \frac{d\phi}{dt}$$

$$i = \frac{\ell}{R}$$

$$i = \frac{\pi (a^2 - b^2) B_0 \omega \cos \omega t}{R}$$

$$R = \rho \times 2\pi (a + b)$$

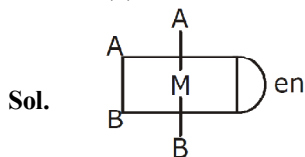
$$\therefore i_{\max} = \frac{1}{2\ell} (a - b) B_0 \omega = 1 \text{ Amp}$$

- 31.** (D)
Sol. $4 \text{ KCN} \cdot \text{Fe}(\text{CN})_2$ is complex salt & it exists as $\text{K}_4[\text{Fe}(\text{CN})_6]$.
- 32.** (C)
Sol. It will give the test of SO_4^{2-} ion in aqueous solution because it is ionizable part which complex will produce. Complex exists as $[\text{Cu}(\text{NH}_3)_4]^{++}$ and SO_4^{2-} ions.
- 33.** (C)
Sol. An ambidentate ligand can donate lone pair of electrons from two atoms but in a complex only one of them donate lone pair of electrons. e.g. $-\text{CN}$ and $-\text{NC}$.
- 34.** (A)
Sol. Double salts are generally made up of two or more salts, when dissolve in water breaks into individual ions, by losing their identity.
- 35.** (A)
Sol. Fajan rule
 Acidic strength \propto polarising power
- $$\begin{matrix} 7+ & 6+ & +5 \\ \text{Cl}_2\text{O}_2 & > & \text{SO}_3 & > & \text{P}_4\text{O}_{10} \end{matrix}$$
- $$\begin{matrix} +6 & 5+ & +4 \\ \text{SO}_3 & > & \text{N}_2\text{O}_5 & > & \text{CO}_2 \end{matrix}$$
- $$\text{Al}_2\text{O}_3 > \text{MgO} > \text{Na}_2\text{O}$$
- 36.** (D)
Sol. Greater the no. of ions, more will be electrical conductance.
- | Equivalent | Conductance |
|------------|--|
| (1) 229 | $[\text{Pt}(\text{NH}_3)_4\text{Cl}_2]\text{Cl}_2$ |
| (2) 97 | $[\text{Pt}(\text{NH}_3)_3\text{Cl}_3]\text{Cl}$ |
| (3) 404 | $[\text{Pt}(\text{NH}_3)_5\text{Cl}]\text{Cl}_3$ |
| (4) 523 | $[\text{Pt}(\text{NH}_3)_6]\text{Cl}_4$ |
- 37.** (C)
Sol. $[\text{Fe}(\text{CN})_6]^{3-}$ & $[\text{Fe}(\text{CN})_6]^{4-}$
 $\text{Fe}(\text{III}) \rightarrow 3d^5$ $\text{Fe}(\text{II}) \rightarrow 3d^6$
 Since both have strong field ligand therefore electron will pair up on the account of this they have d^2sp^3 hybridization. $[\text{Fe}(\text{CN})_6]^{3-}$ contain one unpaired electron $[\text{Fe}(\text{CN})_6]^{4-}$ do not contain unpaired electron thus it will be diamagnetic same structure but different magnetic character.
- 38.** (B)
Sol. If Δ_0 is lower than pairing energy pairing will not take place & system will have $t_{2g}^3 e_g^1$.
- 39.** (C)
Sol. $[\text{CrF}_6]^{4-} \xrightarrow{\text{Weak field ligand}} \text{Cr}(\text{II}) \longrightarrow d^4$
 = 4 unpaired electron
 $[\text{MnF}_6]^{4-} \xrightarrow{\text{Weak field ligand}} \text{Mn}(\text{II}) \longrightarrow d^5$
 = 5 unpaired electron
 $[\text{Cr}(\text{CN})\text{F}_6]^{4-} \xrightarrow{\text{Strong field ligand}} \text{Cr}(\text{II}) \longrightarrow d^4$
 = 2 unpaired electron
 $[\text{Mn}(\text{CN})\text{F}_6]^{4-} \xrightarrow{\text{Strong field ligand}} \text{Mn}(\text{II}) \longrightarrow d^5$
 = 1 unpaired electron

40. (A)

Sol. In this no. of unpaired electron is 2 therefore magnetic moment will be $\sqrt{n(n+2)} = \sqrt{2(2+2)} = \sqrt{8}$ B.M.

41. (B)



Because in this compound there is no plane of symmetry it will show optical isomerism.

42. (B)

Sol. Mo is catalytic promoter & it improves catalytic properties of catalyst.

43. (D)

Sol. Cu, Ag, Au, are coinage metals, used in coin in older days.

44. (A)

Sol. Mn^{2+} acts as auto catalyst. Initially its amount is low & hence decolourisation is slow but later on it becomes fast.

45. (B)

Sol. $AuCl_3 + SnCl_2 \rightarrow SnCl_4 + Au$
Colloidal solution
(Purple of cassius)

46. (C)

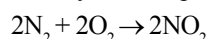
Sol. Conceptual

47. (C)

Sol. Due to attained noble gas configuration of xenon cerium show +4 oxidation state.

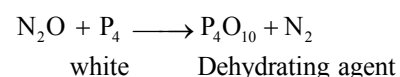
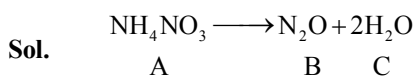
48. (B)

Sol. At very low temperature N_2 doesn't react; but at high temperature NO_2 gas is formed.



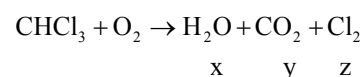
Exothermic process at high temperature.

49. (A)

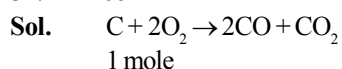


50. (B)

Sol. $CHCl_3 + AgNO_3 \rightarrow$ No ppt.
used as anaesthetic



51. 95



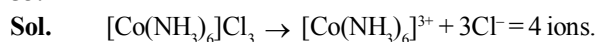
$$\lambda \times \frac{1}{12} = \frac{w_{CO}}{28} + \frac{w_{CO_2}}{44}$$

$$\frac{w_{CO}}{28} + \frac{w_{CO_2}}{44}$$

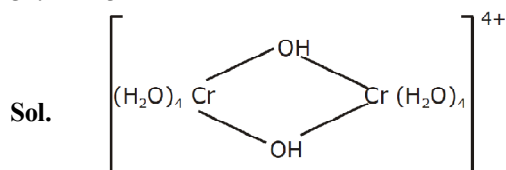
52. 7

Sol. Actinides show +7 maximum oxidation state.

53. 4



54. 3



Oxidation state of Cr is +3.

55. 5

Sol. Its EAN rule follows

$$36 = 26 - 0 + 2 \times 10 = 2x$$

$$x = 5$$

So 5 CO groups should be attached.

56. 5

Sol. $Fe(CO)_x$ follows EAN rule

$$36 = 26 - 0 + 2x \quad 10 = 2x$$

$$x = 5$$

57. 35

Sol. $EAN = 26 + 1 + 8 = 35.$

58. 2

Sol. Let the oxidation number of Fe = x

$$4 + x + (-1) \times 5 + (-1) = 0$$

$$x = +2$$

59. 15

Sol. Theoretically no. of geometrical isomers for octahedral complex [mabcdef] are 15.

60. 18

Sol. 100% – 24 carat

$$1\% = \frac{24}{100}$$

$$75\% = \frac{24}{100} \times 75 = 18$$

61. (A)

Sol.
$$I = \int \frac{\sec x \cos ecx}{2 \cot x - \sec x \cos ecx} dx = \int \frac{dx}{2 \cos^2 x - 1}$$

$$= \int \sec 2x dx = \frac{1}{2} \ln |\sec 2x + \tan 2x| + C$$

62. (C)

Sol.
$$I = \int_2^4 \frac{\log x^2}{\log x^2 + \log(x^2 - 12x + 36)} dx$$

$$I = \frac{2}{2} \int_2^4 \frac{\log x}{\log x + \log(6-x)} dx \quad \dots(i)$$

$$I = \int_2^4 \frac{\log(6-x)}{\log(6-x) + \log x} dx \quad \left\{ \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right\} \quad \dots(ii)$$

Equation (i) & (ii) gives

$$2I = \int_2^4 \frac{\log x + \log(6-x)}{\log x + \log(6-x)} dx = \int_2^4 dx = 2$$

Hence $I = 1$



63. (B)

Sol.
$$\frac{3x-4}{3x+4} = t$$

$$3x-4 = 3xt+4t$$

$$x = \frac{4t+4}{3(1-t)}$$

$$f(t) = \frac{4t+4}{3(1-t)} + 2$$

$$f(x) = \frac{4x+4}{3(1-x)} + 2 = \frac{4(x-1)+8}{3(1-x)} + 2$$

$$f(x) = 2 - \frac{4}{3} - \frac{8}{3(x-1)} = \frac{2}{3} - \frac{8}{3(x-1)}$$

$$\int f(x) dx = \frac{2}{3}x - \frac{8}{3} \ln|x-1| + c$$

64. (B)

Sol. Let $\ln x = t^2$

$$x = e^{t^2} \Rightarrow dx = e^{t^2} \cdot 2t dt$$

$$2 \int_1^2 t^2 \cdot e^{t^2} \cdot dt = \int_1^2 t(2te^{t^2}) dt$$

$$= (t e^{t^2})_1^2 - \int_1^2 e^{t^2} dt = 2e^4 - e - \alpha$$

65. (B)

Sol. Put $\tan^{-1}(\sec x + \cos x) = t$

$$\frac{1}{1 + (\sec x + \cos x)^2} (\sec x \tan x - \sin x) dx = dt$$

$$\frac{\cos^2 x}{\cos^2 x + (1 + \cos^2 x)^2} \sin x \left(\frac{1}{\cos^2 x} - 1 \right) dx = dt$$

$$\frac{\cos^2 x \sin x \frac{(1 - \cos^2 x)}{\cos^2 x}}{\cos^2 x + 1 + \cos^4 x + 2 \cos^2 x} dx = dt$$

$$\frac{\sin^3 x}{\cos^4 x + 3 \cos^2 x + 1} dx = dt$$

$$I = \int \frac{dt}{t} \Rightarrow \log |t| + c$$

$$\Rightarrow \log |\tan^{-1}(\sec x + \cos x)| + c$$

66. (C)

Sol. $I_2 = \int_0^1 \frac{\tan^{-1} x}{x} dx$, $x = \tan \theta$

$$\Rightarrow I_2 = \int_0^{\pi/4} \frac{2\theta}{\sin 2\theta} d\theta = \frac{1}{2} \int_0^{\pi/2} \frac{x}{\sin x} dx = \frac{1}{2} \cdot I_1$$

$$\Rightarrow \frac{I_1}{I_2} = 2$$

67. (A)

Sol. With $\alpha = \tan^{-1} \frac{5}{12}$, $\sin \alpha = \frac{5}{13}$ and $\cos \alpha = \frac{12}{13}$

$$\int \frac{dx}{12 \sin x + 5 \cos x} = \int \frac{dx}{13 \sin(x + \alpha)}$$

$$= \frac{1}{13} \int \frac{\frac{1}{2} \sec^2 \frac{x + \alpha}{2}}{\tan \frac{x + \alpha}{2}} dx$$

$$= \frac{1}{13} \log \left| \tan \left(\frac{x}{2} + \frac{1}{2} \tan^{-1} \frac{5}{12} \right) \right| + c$$

68. (D)

Sol. Integrating by parts, the given integral is equal to

$$x \tan^{-1} \sqrt{\sqrt{x} - 1} \Big|_1^{16} - \int_1^{16} \frac{x}{\sqrt{x}} \frac{1}{4\sqrt{x}\sqrt{\sqrt{x} - 1}} dx$$

$$= \frac{16}{3} \pi - \frac{1}{4} \int_1^{16} \frac{dx}{\sqrt{\sqrt{x} - 1}}$$

$$= \frac{16}{3} \pi - \frac{1}{4} \int_0^{\sqrt{3}} \frac{4t(1+t^2)}{t} dt \quad (\sqrt{x} = 1+t^2)$$

$$= \frac{16}{3} \pi - (\sqrt{3} + \sqrt{3}) = \frac{16}{3} \pi - 2\sqrt{3}$$

69. (C)

Sol. $\int \sin^4 x \cos^4 x \, dx = \frac{1}{16} \int \sin^4 2x \, dx = \frac{1}{16} \int \left(\frac{1 - \cos 4x}{2} \right)^2 dx = \frac{1}{64} \int (1 + \cos^2 4x - 2\cos 4x) \, dx$

$$= \frac{1}{64} \int dx - \frac{1}{32} \int \cos 4x \, dx + \frac{1}{64} \int \left(\frac{1 + \cos 8x}{2} \right) dx$$
$$= \frac{x}{64} - \frac{\sin 4x}{128} + \frac{1}{64} \times \frac{x}{2} + \frac{1}{128} \frac{\sin 8x}{8} + C$$
$$= \frac{3}{128} x - \frac{1}{128} \sin 4x + \frac{\sin 8x}{128 \times 8} + C$$
$$= \frac{1}{128} \left[3x - \sin 4x + \frac{\sin 8x}{8} \right] + C$$

70. (A)

Sol. $I = \int_0^{\pi} \log(1 + \cos x) \, dx$

$$= \int_0^{\pi} \log \left(2 \cos^2 \frac{x}{2} \right) dx$$
$$= \int_0^{\pi} \log 2 \, dx + \int_0^{\pi} \log \cos^2 \frac{x}{2} \, dx$$
$$= \pi \log 2 + 2 \int_0^{\pi} \log \cos \frac{x}{2} \, dx$$
$$= \pi \log 2 + 4 \int_0^{\pi/2} \log \cos x \, dx$$
$$= \pi \log 2 + 4 \int_0^{\pi/2} \log \cos(\pi/2 - x) \, dx$$
$$= \pi \log 2 + 4 \int_0^{\pi/2} \log \sin x \, dx$$
$$= \pi \log 2 + 4k$$

PE

71. (C)

Sol. $I = \int \frac{\sin x + \cos x}{\sqrt{\cos x \sin x}} \, dx$

Put $\sin x - \cos x = t$, so that $(\sin x + \cos x) \, dx = dt$ and $1 - 2 \sin x \cos x = t^2$

$$\therefore I = \sqrt{2} \int \frac{dt}{\sqrt{1-t^2}} = \sqrt{2} \sin^{-1}(t) + C$$
$$= \sqrt{2} \sin^{-1}(\sin x - \cos x) + C$$

72. (C)

Sol. $f(x) = \frac{\sin x}{x}$ is a decreasing function and

$$\frac{\sin x}{x} > 0 \text{ for all } x \text{ in } (0, \pi)$$

Since $\sin x < x < \tan x$

$$\Rightarrow \frac{\sin(\sin x)}{\sin x} > \frac{\sin x}{x} > \frac{\sin(\tan x)}{\tan x} \text{ for } \frac{\pi}{6} < x < \frac{\pi}{3}$$

$$\therefore I_2 > I_1 > I_3$$

73. (B)

Sol. Write $I = \int \frac{x^2 dx}{x^3 \sqrt{1-x^3}}$ and $1-x^3 = t^2$, so that $-3x^2 dx = 2t dt$ and

$$I = \int \frac{(-2/3)t dt}{(1-t^2)t} = -\frac{2}{3} \int \frac{dt}{1-t^2}$$

$$= -\frac{2}{3} \left(\frac{1}{2} \right) \log \left| \frac{1-t}{1+t} \right| + C = 3 \log \left| \frac{\sqrt{1-x^3}+1}{\sqrt{1-x^3}-1} \right| + C$$

74. (A)

Sol. We can write

$$I = I_1 + I_2 + I_3$$

$$\text{where } I_1 = \int_{-3}^2 |x+1| dx \text{ etc.}$$

Put $x+1 = t$, so that

$$I_1 = \int_{-2}^3 |t| dt = \int_{-2}^0 (-t) dt + \int_0^3 t dt$$

$$= -\frac{1}{2}t^2 \Big|_{-2}^0 + \frac{1}{2}t^2 \Big|_0^3 = \frac{13}{2}$$

$$\text{Similarly, } I_2 = I_3 = \frac{9}{2}$$

$$\text{Thus, } I = \frac{31}{2}$$

75. (B)

Sol. Put $1+x^3 = t^2$, $3x^2 dx = 2t dt$, so that

$$I = \int \frac{(t^2-1)}{t} \cdot t dt = \frac{2}{3} \left(\frac{t^3}{3} - t \right) + C$$

$$= \frac{2}{9} (1+x^3)^{3/2} - \frac{2}{3} (1+x^3)^{1/2} + C$$

76. (B)

Sol.
$$\int_0^1 \frac{dx}{\sqrt{1-(x \sin \alpha)^2}} = \left[\frac{\sin^{-1}(x \sin \alpha)}{\sin \alpha} \right]_0^1$$
$$= \frac{\sin^{-1}(\sin \alpha) - 0}{\sin \alpha} = \frac{\pi - \alpha}{\sin \alpha}$$

(\because for $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$ $\sin^{-1} \sin \alpha = \pi - \alpha$)

$$= \left| \frac{\pi - \alpha}{\sin \alpha} \right| \text{ (For given interval } \frac{\pi - \alpha}{\sin \alpha} = \left| \frac{\pi - \alpha}{\sin \alpha} \right|)$$

77. (C)

Sol.
$$I = \int \frac{1}{x} \frac{1-x}{\sqrt{1-x^2}} dx$$
$$= \int \frac{dx}{x\sqrt{1-x^2}} - \int \frac{dx}{\sqrt{1-x^2}}$$

In the first integral, put $x = \frac{1}{t}$, so that

$$I = \int \frac{(-1/t^2) dt}{\frac{1}{t} \sqrt{1-1/t^2}} - \sin^{-1} x = - \int \frac{dt}{\sqrt{t^2-1}} - \sin^{-1} x$$
$$= -\log |t + \sqrt{t^2-1}| - \sin^{-1} x + C$$
$$= -\log \left| \frac{1}{x} + \frac{\sqrt{1-x^2}}{x} \right| - \sin^{-1} x + C = \log |x| - \log |1 + \sqrt{1-x^2}| - \sin^{-1} x + C$$

78. (A)

Sol.
$$g(x + \pi) = \int_0^{x+\pi} \cos^4(t) dt$$
$$= \int_0^x \cos^4(t) dt + \int_x^{x+\pi} \cos^4 t dt \quad \dots (3)$$
$$= \int_0^x \cos^4 t dt + \int_0^\pi \cos^4 t dt \quad (\because \cos^4 t \text{ is periodic function})$$
$$= g(x) + g(\pi)$$

79. (A)

Sol. Put $t = \frac{1}{2}(x - \alpha + x - \beta)$

$$= x - \frac{1}{2}(\alpha + \beta), \text{ so that}$$

$$(x - \alpha)(\beta - x) = \left[t + \frac{1}{2}(\alpha + \beta) - \alpha \right]$$

$$\left[\beta - \frac{1}{2}(\alpha + \beta) - t \right]$$

$$= \frac{1}{4}(\beta - \alpha)^2 - t^2$$

$$\therefore I = \sin^{-1} \left(\frac{2t}{\beta - \alpha} \right) + C = \sin^{-1} \left(\frac{2x - \alpha - \beta}{\beta - \alpha} \right) + C$$

80. (B)

Sol. We can write

$$I = I_1 + I_2$$

$$\text{where } I_1 = \int_{-\pi}^{\pi} \frac{2x}{1 + \cos^2 x} dx = 0$$

$$\text{and } I_2 = 4 \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}$$

$$= 4 \int_0^{\pi} \frac{(\pi - x) \sin(\pi - x)}{1 + (\cos(\pi - x))^2} dx$$

$$= 4\pi \int_0^{\pi} \frac{dx}{1 + \cos^2 x} - I_2$$

$$\Rightarrow I_2 = 2\pi \int_0^{\pi} \frac{\operatorname{cosec}^2 x}{\cos^2 x + \cot^2 x} dx$$

$$= 2\pi \int_0^{\pi} \frac{\operatorname{cosec}^2 x dx}{1 + 2 \cot^2 x}$$

$$= \frac{-2\pi}{\sqrt{2}} \tan^{-1}(\sqrt{2} \cot x) \Big|_0^{\pi}$$

$$= -\sqrt{2} \left(-\frac{\pi}{2} - \frac{\pi}{2} \right) = \sqrt{2}\pi$$

81. 5

Sol. $\cos^5 x + \sin^5 x = (\cos x + \sin x) [\cos^4 x - \cos^3 x \sin x + \cos^2 x \sin^2 x - \cos x \sin^3 x + \sin^4 x]$

$$= (\cos x + \sin x) [(\cos^2 x + \sin^2 x)^2 - \cos^2 x \sin^2 x - \cos x \sin x (\cos^2 x + \sin^2 x)]$$

$$= (\cos x + \sin x) (1 - \cos x \sin x - \cos^2 x \sin^2 x)$$

$$\therefore I =$$

$$\int \frac{\sin x + \cos x}{(1 + 2 \sin x \cos x)(1 - \cos x \sin x - \cos^2 x \sin^2 x)} dx$$

Put $\sin x - \cos x = t$, so that $(\cos x + \sin x) dx = dt$, and $t^2 = 1 - 2 \sin x \cos x$

Thus

$$\begin{aligned} I &= \int \frac{dt}{(2-t^2) \left[1 - \frac{t^2-1}{2} - \frac{1}{4}(t^2-1)^2 \right]} \\ &= 4 \int \frac{dt}{(2-t^2)(5-t^4)} \\ &= 4 \int \left[\frac{1}{2-t^2} + \frac{1}{2-\sqrt{5}} \frac{1}{2\sqrt{5}} \frac{1}{\sqrt{5}-t^2} + \frac{1}{2+\sqrt{5}} \frac{1}{2\sqrt{5}} \frac{1}{\sqrt{5}+t^2} \right] dt \\ &= \sqrt{2} \log \left| \frac{\sqrt{2}+t}{\sqrt{2}-t} \right| \frac{1}{2-\sqrt{5}} \frac{1}{5^{3/4}} \log \left| \frac{5^{1/4}+t}{5^{1/4}-t} \right| + \frac{1}{2+\sqrt{5}} \frac{2}{5^{3/4}} \tan^{-1} \left(\frac{t}{5^{1/4}} \right) + C \end{aligned}$$

where $t = \sin x - \cos x$

82. 16

Sol. $\int_1^4 \frac{2e^{\sin x^2}}{x} dx = f(x^2) \Big|_1^4 = f(16) - f(1)$

$\Rightarrow k = 16$

83. 12

Sol. $f(x) = \int \frac{3x+2}{\sqrt{x-9}} dx$ put $x-9 = t^2$

$$= 2 \int (29 + 3t^2) dt$$

$f(x) = 2(29\sqrt{x-9} + (x-9)^{3/2}) + C$

$\therefore f(10) = 60 \Rightarrow C = 0$

$f(x) = 2(29 + (x-9)^{3/2})$

$f(13) = 132 \Rightarrow$ sum of digits = 6
 $2(\text{sum of digits}) = 12$

84. 158

Sol. $f(x) = x + \lambda_1 x^2 + \lambda_2 x$, where $\lambda_1 = \int_0^1 z f(z) dz$, $\lambda_2 = \int_0^1 z^2 f(z) dz$

on solving for λ_1 we have $9\lambda_1 - 4\lambda_2 = 4 \dots (i)$

and for λ_2 we have $15\lambda_2 - 4\lambda_1 = 5 \dots (ii)$

from (i) and (ii) $\lambda_1 = \frac{80}{119}$, $\lambda_2 = \frac{61}{119}$

$f(x) = x + \frac{80x^2}{119} + \frac{61}{119}x$

$f(x) = \frac{80x^2}{119} + \frac{61}{119}x$

$f(10) = 157.98 \simeq 158$

85. 10

Sol. $I = \int \frac{\sqrt{4+x^2}}{x^6} dx$ Put $x = 2 \tan \theta \Rightarrow dx = 2 \sec^2 \theta d\theta$

$$\Rightarrow I = \int \frac{2 \sec \theta \cdot 2 \sec^2 \theta d\theta}{2^6 \tan^6 \theta} = \frac{1}{2^4} \int \frac{\cos^3 \theta d\theta}{\sin^6 \theta}$$

$$= \frac{1}{2^4} \int \left(\frac{1 - \sin^2 \theta}{\sin^6 \theta} \right) \cos \theta d\theta \quad \text{put } \sin \theta = t \Rightarrow \cos \theta d\theta = dt$$

$$= \frac{1}{2^4} \int \frac{dt}{t^6} - \int \frac{1}{t^4} dt = \frac{1}{16} \left[\frac{1}{3t^3} - \frac{1}{5t^5} \right] + C = \frac{1}{16} \left(\frac{5 \sin^2 \theta - 3}{15 \sin^5 \theta} \right) + C$$

but $\tan \theta = \frac{x}{2}$ So $\sin \theta = \frac{x}{\sqrt{4+x^2}}$

$$= \frac{1}{16} \left(\frac{\frac{5x^2}{x^2+4} - 3}{\left(\frac{x}{\sqrt{4+x^2}}\right)^5} \right) + C = \frac{1}{16} \left(\frac{(2x^2 - 12)(x^2 + 4)^{3/2}}{15x^5} \right) + C = \frac{1}{120} \left(\frac{(x^2 + 4)^{3/2} (x^2 - 6)}{x^5} \right) + C$$

86. 7

Sol. $\int_0^{\pi/4} (\cos 2\theta)^{3/2} \cos \theta d\theta = \int_0^{\pi/4} (1 - 2 \sin^2 \theta)^{3/2} \cos \theta d\theta$

Put $\sin \theta = t$

$$I = \int_0^{1/\sqrt{2}} (1 - 2t^2)^{3/2} dt$$

again put $\sqrt{2}t = \sin z$

87. 2

Sol. $\int e^{\sin x} \cdot \frac{x \cos^3 x - \sin x}{\cos^2 x} dx = \int e^{\sin x} (x \cos x - \tan x \sec x) dx =$

$$\int e^{\sin x} \cdot x \cos x dx - \int e^{\sin x} \tan x \sec x dx$$

(by parts integration)

$$= x e^{\sin x} - \int 1 \cdot e^{\sin x} dx - [e^{\sin x} \sec x - \int e^{\sin x} \cos x \cdot \sec x dx] + C$$

$$= e^{\sin x} (x - \sec x) + C \quad \therefore f(x) = x - \sec x$$

$$f\left(\frac{\pi}{3}\right) = \frac{\pi}{3} - 2$$

88. 2

Sol. Putting $x = a \sin \theta$

$$I = \int_0^{\pi/2} \frac{\theta^2 \cos^2 \theta d\theta}{c^2 - a^2 \sin^2 \theta}$$

$$= a^2 \int_0^{\pi/2} \left(\frac{1}{a^2} + \frac{\left(1 - \frac{c^2}{a^2}\right)}{c^2 - a^2 \sin^2 \theta} \right) d\theta$$

$$\begin{aligned}
&= \frac{\pi}{2} + (a^2 - c^2) \int_0^{\pi/2} \frac{\sec^2 \theta d\theta}{c^2 \sec^2 \theta - a^2 \tan^2 \theta} \\
&= \frac{\pi}{2} + (a^2 - c^2) \int_0^{\infty} \frac{dt}{c^2(1+t^2) - a^2 t^2} \quad (\tan \theta = t) \\
&= \frac{\pi}{2} - \frac{\pi\sqrt{c^2 - a^2}}{2c} \\
&\Rightarrow \lambda = 2
\end{aligned}$$

89. 16

Sol. $g(x) = \int \frac{1 + 2\cos x}{(\cos x + 2)^2} dx$; $g(x) = \int \frac{\cos x(\cos x + 2) + \sin^2 x}{(\cos x + 2)^2} dx$

$$\begin{aligned}
g(x) &= \int \cos x \cdot \frac{1}{(\cos x + 2)} dx + \int \frac{\sin^2 x}{(\cos x + 2)^2} dx \\
&= \frac{1}{(\cos x + 2)} \cdot \sin x - \int -\frac{(-\sin x)}{(\cos x + 2)^2} \cdot \sin x dx + \int \frac{\sin^2 x}{(\cos x + 2)^2} dx
\end{aligned}$$

$$g(x) = \frac{\sin x}{(\cos x + 2)} + C$$

$$g(0) = 0 \Rightarrow C = 0$$

$$g(x) = \frac{\sin x}{\cos x + 2}$$

$$g\left(\frac{\pi}{2}\right) = \frac{1}{2} ; 32g\left(\frac{\pi}{2}\right) = 16$$



90. 3

Sol. Let $I_n = \int_0^{\pi/2} x^n \sin x dx$

Integrate by parts and choose $\sin x$ as the second function.

$$I_n = 0 + n \int_0^{\pi/2} x^{n-1} \cos x dx$$

Again integrating by parts, we get

$$I_n = n \left(\frac{\pi}{2}\right)^{n-1} - n(n-1) I_{n-2}$$

R.H.S. contain $\pi^2 \therefore$ put $n = 3$

$$I_3 = 3 \left(\frac{\pi}{2}\right)^2 - 3 \cdot 2 I_1 = \frac{3\pi^2}{4} - 6 \int_0^{\pi/2} x \sin x dx$$

$$= \frac{3}{4} (\pi^2 - 8) \text{ which is true.}$$

Hence, $n = 3$.