JEE MAIN ANSWER KEY & SOLUTION

PAPER CODE :- PART TEST-4
CLASS-XI

ANSWER KEY

PHYSICS

1.	(A)	**2.**	(C)	**3.**	(B)	**4.**	(B)	**5.**	(C)	**6.**	(D)	**7.**	(D)
8.	(B)	**9.**	(A)	**10.**	(C)	**11.**	(C)	**12.**	(C)	**13.**	(C)	**14.**	(B)
15.	(C)	**16.**	(B)	**17.**	(C)	**18.**	(B)	**19.**	(B)	**20.**	(C)	**21.**	3
22.	140	**23.**	2	**24.**	28.9	**25.**	3	**26.**	9	**27.**	5	**28.**	60
29.	64	**30.**	1										

CHEMISTRY

31.	(C)	**32.**	(A)	**33.**	(C)	**34.**	(A).	**35.**	(C)	**36.**	(B)	**37.**	(B)
38.	(D)	**39.**	(B)	**40.**	(C)	**41.**	(C)	**42.**	(C)	**43.**	(A)	**44.**	**(A)**
45.	(D)	**46.**	(C)	**47.**	(C)	**48.**	(D)	**49.**	(D)	**50.**	(D)	**51.**	1
52.	48	**53.**	64	**54.**	6	**55.**	200	**56.**	1	**57.**	0.16	**58.**	68
59.	9	**60.**	5										

MATHEMATICS

61.	(A)	**62.**	(A)	**63.**	(B)	**64.**	(A)	**65.**	(A)	**66.**	(C)	**67.**	(D)
68.	(A)	**69.**	(D)	**70.**	(D)	**71.**	(D)	**72.**	(D)	**73.**	(C)	**74.**	(D)
75.	(B)	**76.**	(A)	**77.**	(A)	**78.**	(A)	**79.**	(D)	**80.**	(A)	**81.**	4
82.	2	**83.**	64	**84.**	38	**85.**	40	**86.**	3	**87.**	9	**88.**	11
89.	11	**90.**	8										

PE

SOLUTIONS

PHYSICS

1. (A)

Sol. $PV = RT \Rightarrow PdV = RdT$

$\therefore$ Coefficient of volume expansion

$$= \frac{1}{V}\frac{dV}{dT} = \frac{R}{PV} = \frac{1}{T}$$

2. (C)

Sol. $Q = m.c. \Rightarrow c = \dfrac{Q}{m.\Delta\theta}$; when $\Delta\theta = 0 \Rightarrow c = \infty$

3. (B)

Sol. $v_{rms} \propto \sqrt{T}$ [T = temperature in Kelvin]

$T_1 = 27 + 273 = 300\,K$

$T_2 = 4 \times 27 + 273 = 381\,K$

$$\therefore v_2 = \sqrt{\frac{381}{300}} \times 500 \simeq 560\,m/s$$

4. (B)

Sol. Internal energy change is

$\Delta U = n\,C_v\,\Delta T$

$$= n\frac{5}{2}R(T-0)$$

$$= \frac{5}{2}nRT$$

5. (C)

Sol. For anisotropic material

$\gamma = \alpha + 2\alpha + 3\alpha = 6\alpha$

6. (D)

Sol. Heat released by water

$\Delta Q = 80 \times 1 \times 30 = 2400$ cal (i)

Mass of Ice melt

$2400 \times m \times 80$ $[\Delta Q = mL]$

$$\therefore m = \frac{2400}{80} = 30\,gm$$

7. (D)

Sol. $\dfrac{\Delta Q}{W} = \dfrac{C_P}{R}$

$$C_P = \left(\frac{f}{2} + 1\right)R$$

(f = degree of produce)

$= 4R$

$\therefore \Delta Q = 4W = 120\,J$

8. (B)

Sol. Heat is a path function. Heat transfer depends on process. Hence heat transfer is different for different paths between same initial & final status.

9. (A)

Sol. Excess pressure is directed towards centre of curvature and inversely proportional to radius of curvature.

10. (C)

Sol. $y_S = 3\alpha_S$

$\alpha_S = \frac{Y}{3}$ given

So, $y_S = 3 \times \frac{y}{3} = y$

11. (C)

Sol. Relation between the two scales

$$\frac{t-10}{80-10} = \frac{F-32}{180}$$

$$F = \frac{18}{7}(t - 10) + 32$$

12. (C)

Sol. $PV = \frac{1}{3} m_o N v_{rms}^2$

$(2P)(2V) = \frac{1}{3} m_o N v'^2_{rms}$

$\varpi'_{rms} = 2v_{rms} = 2v$

13. (C)

Sol. Mass of liquid inside the capillary $= \pi r^2 h d$

$$= (\pi r h d).r$$

since, hr = constant

$\therefore$ mass of liquid inside α r

14. (B)

Sol. Work done is area under the curve

$W = -P_o V_o$

$= -8 \times (10^5 \times 1.01) \times 7 \times 10^{-3}$

$W = -56 \times 1.01 \times 10^2$ Joule

$W = -5656$ Joule

15. (C)

Sol. $\alpha = 10^{-5}/^\circ C$ $\frac{\Delta \ell}{\ell} = 0.10\%$, $\Delta T = 100^\circ C$

$\therefore \frac{\Delta \ell}{\ell} = \alpha \Delta\theta$ and $\frac{\Delta V}{V} = \gamma. \Delta\theta = 3\alpha.\Delta\theta$

16. (B)

Sol. heat release by water = m s d θ

= 300 × 1 × 25

= 7500 Cal.

amount of Ice melts from this heat

dQ = mL

$$m = \frac{dQ}{L} = \frac{7500}{80} = 93.75 \text{ g.}$$

17. (C)

Sol. $$Y = \frac{F\ell}{A\Delta\ell} = \frac{2Mg(1-\cos\theta)L}{\pi\frac{d^2}{4}\Delta\ell}$$

$$[\because \frac{Mv^2}{2} = Mg\ell(1-\cos\theta)$$

$$\Rightarrow \frac{Mv^2}{\ell} = 2Mg(1-\cos\theta)]$$

$$1 - \cos\theta = \frac{Y\pi d^2\Delta\ell}{8Mg\ell} \Rightarrow \cos\theta = 1 - \frac{Y\pi d^2\Delta\ell}{8Mg\ell}$$

18. (B)

Sol. Time period of oscillation of piston is $T \propto \frac{1}{\sqrt{r}}$ where $\gamma = C_p/C_v$ adiabatic exponent

$\gamma_{mono} = 5/3$; $\gamma_{di} = 7/5$; $\gamma_{poly} = 4/3$

$\therefore \gamma_{mono} > \gamma_{di} > \gamma_{poly}$

19. (B)

Sol. When the pressure is first increased by 10% it becomes (110/100) P. when it is decreased by 10% from there, the pressure becomes

$$\frac{110\times90}{100\times100} P = \frac{99}{100} P$$

Thus the pressure decreases by 1% volume is increased by nearly 1%

20. (C)

Sol. $$\frac{dQ}{dt} = KA\frac{dT}{dx} \Rightarrow \frac{1}{A}\frac{dQ}{dt} = K\frac{dT}{dx}$$

$\frac{1}{A}\frac{dQ}{dt}$ is same so $\frac{dT}{dx}$ is smaller for higher K.

21. 3

Sol. Rate of increase in Fahrenheit = 1.8 × Rate of increase in Celsius scale

Ans. 3° F/sec

22. 140

Sol. $$\frac{\Delta Q}{W} = \frac{nC_p\Delta T}{nR\Delta T}$$

$$\Rightarrow Q = \frac{C_P}{R} W$$

$$= \frac{7}{2} \times 20$$

= 140 J

23. 2

Sol. $x = \ell_0 \alpha t$

$x = 2$

24. 28.9

Sol. The correct answer is 28.9 cc

Given, d=20 cm

$V=V_0=(1+\gamma\Delta T)=V0(1+3\alpha\Delta T)$

Where γ is coefficient of volumetric expansion.

$\because \gamma=3\alpha$

Volume of sphere $= \frac{4}{3}\pi r^3$

Change in volume=V-V0=3V0a?T=3×43p

$$= 3\times\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times 23\times10^{-6}\times100$$

$$= 3\times\frac{4}{3}\pi\left(\frac{0.2}{2}\right)^3 \times 23\times10^{-6}\times100$$

$= 28.9$ cc(1 cc=10^{-6} m^3)

25. 3

Sol. $\Delta Q = \Delta U + W$

$10 = \Delta U + 5 \Rightarrow \Delta U = 5J$

$\Rightarrow \quad nC_V\Delta T = 5 \quad$ ....(1)

Now, $\Delta Q = nC\Delta T$

$10 = C \times \frac{5}{C_V}$ [from equation (1)]

or $\quad C = 2C_V$

26. 9

Sol. $Q = ms\Delta T + mL$

$= 450$ cal

Ans. $450 \times 4 = 9000$ J

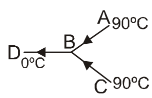
27. 5

Sol. $v_{sound} = \sqrt{\frac{\gamma P}{\rho}}$ and $\gamma = 1 + \frac{2}{f}$

$\therefore \quad 1 + \frac{2}{f} = \frac{v_{sound}^2 \times \rho}{P}$

28. 60

Sol. Let q be the temperature of the junction (say B). Thermal resistance of all the three rods is equal. Rate of heat flow through AB + Rate of heat flow through CB = Rate of heat flow through BD



$\therefore \quad \frac{90-\theta}{R} + \frac{90-\theta}{R} = \frac{\theta - 0}{R}$

Here R = Thermal Resistance

$\therefore \quad 3\theta = 180$ or $\quad \theta = 60ºC$

29. 64

Sol. $\text{ratio} = \dfrac{e\sigma(4\pi(2R)^2)(2T)^4}{e\sigma(4\pi R^2)T^4} = 64$

30. 1

Sol. $\dfrac{40-T}{R_H/2} = \dfrac{T-20}{R_H/2} + \dfrac{T-0}{R_H/4}$

$T = 15°C$

$\dfrac{T-0}{R_H/4} = i_H \Rightarrow i_H = 6$ J/s

Heat supplied $= 6 \times 5.6 \times 10^4 = 3.36 \times 10^5$ J In 5.6×10^4s. amount of ice $mL_f = 3.36 \times 10^5$

PE

31. (C)

Sol.
$$K_{aq} = \frac{K_f}{K_b}$$
$\Rightarrow K_f = K_{aq} \times K_b = 11.25 \times 10^{-4}$
$= 1.5 \times 7.5 \times 10^{-4} = 11.25 \times 10^{-4}$
$= 1.125 \times 10^{-5}$

32. (A)

Sol. $Ag^+ + 2NH_3 \rightleftharpoons Ag(NH_3)_2+$
$$K_c = \frac{\left[Ag(NH_3)_2^+\right]}{\left[Ag^+\right][NH_3]^2} = \frac{10^{-1}}{(10^{-1})\times(10^{3})^2} = 10^{-6}$$

33. (C)

Sol. The value of K is independent of initial concentration of reactants(s) and product(s)

34. (A).

Sol. In the presence of catalyst, an alternative pathway with lower activation energy is made available. More collisions are successful because less energy is required for success. After the reaction, the catalyst can be recovered after the reaction is over and used again and again.

35. (C)

Sol.

P	$\rightleftharpoons$	Q	+	R
2		0		0
$2(1-\alpha)$		2α		2α

At equilibrium, 0.5 moles was dissociated so, the remaining moles of P is 1.5
$\therefore 2(1-\alpha) = 1.5$
$2 - 2\alpha = 1.5$
$-2\alpha = 1.5 - 2$
$-2\alpha = -0.5$
$\alpha = 0.5/2 = 0.25.$

36. (B)

Sol.

	$PCl_5(g)$	$\rightleftharpoons$	$PCl_3(g)$	+	$Cl_2(g)$
Intial	8		0		0
At eqm.	$(8-8\alpha)$		8α		8α

Total moles $= 8 - 8\alpha + 8\alpha + 8\alpha$
$$= 8 + 8\alpha = 8 + 8 \times \frac{25}{100} = 10$$
Degree of dissociation $= 25\%$
$$X_{PCl_5} = \frac{8-8\alpha}{10}, X_{PCl_3} = \frac{8\alpha}{10}, X_{Cl_2} = \frac{8\alpha}{10}$$
$$X_{PCl_5} = \frac{6}{10} \times P,\ P_{Cl_2} = \frac{2}{10} \times P$$
$$K_P = \frac{P_{Cl_3} \times P_{Cl_2}}{P_{Cl_5}} = \frac{\left(\frac{2}{10}\times P\right)\times\left(\frac{2}{10}\times P\right)}{\left(\frac{6}{10}\times P\right)}$$
$$= \frac{4P}{60} = \frac{P}{15}$$

37. (B)

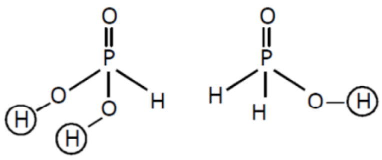
Sol. $x = 1$ and $n = 2$ (NH_4Cl ’! $NH_3 + HCl$)

$d = D - d$

$$\text{or, } d = \frac{D}{2}$$

38. (D)

Sol. Basicity of H_3PO_3 and H_3PO_2 are 2 & 1



39. (B)

Sol. Conc. of HOCl = 0.15 M

$K_a = 9.6 \times 10^{-6}$

$$K_a = \frac{n^2}{0.15}$$

$$9.6 \times 10^{-6} \times \frac{15}{100} = n^2$$

$$\frac{96}{10} \times \frac{1}{10^6} \times \frac{15}{100} = n^2$$

$$\sqrt{\frac{1440}{10 \times 10^8}} = n$$

$\Rightarrow 1.2 \times 10^{-3} = n$ pH = 3 – log(1.2)

3 – 0.079 pH = 2.921

40. (C)

Sol. pH of K_2S solution is more than 7

$K_2S \longrightarrow 2K^+ + S^{2-}$

$S^{2-} + H_2O \longrightarrow K_2S + {-OH}$

Here, H_2S is weak acid and generation of OH will make the solution basic

$\therefore$ pH > 7.

41. (C)

Sol. At equilibrium, $r_f = r_b$

$\therefore K_f[A]_{en} = K_b[B]_{en}$

$[B]_{eq} = K_f Kb^{-1} [A]_{eq}$

42. (C)

Sol. When $Q > K_C$, the reaction will proceed in backward direction to attain equilibrium

43. (A)

Sol. (A - r) ; (B - q) ; (C - p)

(A) Δn_g is +ve so as P is increased, backward shifting will take place.

(B) No change as $\Delta n_g = 0$.

(C) Forward shifting will take place.

44. **(A)**

Sol. $N_2 + O_2 \rightleftharpoons 2NO$

Here, $\Delta n = O$

So, Increase in pressure at equilibrium has no effect on the Reaction. Both, Assertion Reason are true and Reason is correct explanation of Assertion.

45. (D)

Sol. $Li_3Na_3(AlF_6)_2$ solubility = 5 mol L^{-1}

$\therefore K_{Sp}\ (3S)_3\ (3S)^3\ (2S)^2$

$K_{Sp} = 2916\ S^8$

46. (C)

Sol.
$$Ag_2SO_4 \longrightarrow \underset{4S^2}{2Ag^+} + \underset{S}{SO_4^{2-}}$$

$K_{sp} = 4S^3, K_{sp} = 2 \times 10^{-5}$

$S = \sqrt{\dfrac{2\times10^{-5}}{4}} = 0.017\ m/\ell \quad = 1.7 \times 10^{-2}$

$AgBrO_3 \longrightarrow Ag^+ + BrO_3^-$

$K_{sp} = S^2, K_{sp} = 5.5 \times 10^{-5}$

$S = \sqrt{5.5\times10^{-5}} = 7.4 \times 10{-}3\ m/\ell$

47. (C)

Sol. $\alpha\,\% = \sqrt{\dfrac{K_a}{C}} \times 100 \qquad \Rightarrow \sqrt{\dfrac{1.8\times10^{-5}}{0.2}} \times 100 = 0.950$

48. (D)

Sol. Aspirin a weak acid is unionised in acid medium due to common ion effect and completely ionised in alkaline medium.

49. (D)

Sol. Aqueous solution of CuSO4 turns blue litmus red as it gives acidic solution on hydrolysis.

50. (D)

Sol. Congugate base is formed by the removal of one H+ from acid :

$H_2PO_4^- \longrightarrow HPO_4^{2-} + H^+$

51. 1

Sol. $C(s) + CO_2(g) \rightleftharpoons 2CO(g)$

$$K_P = \frac{P_{(P)^2}}{P_{(CO_2)}} = \frac{(2)^2}{4} = \frac{4}{4} = 1$$

52. 48

Sol. $A \rightleftharpoons B$

$$K_c = 2 = \frac{[B]}{[A]} \quad .....(i)$$

$B \rightleftharpoons C$ or, $[B] = 2[A]$

$$K_c = 4 = \frac{[C]}{[B]}$$

From eq. (i)

$$K_c = 4 = \frac{[C]}{2[A]}$$

$$[A] = \frac{[C]}{8} \quad .....(ii)$$

$C \rightleftharpoons D$

$$K^c = \frac{[D]}{[A]} = \frac{6[C]}{[C]/8} = 48$$

53. 64

Sol. $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$

$$K_{c_1} = \frac{[HI]^2}{[HI_2][I_2]} = 64$$

Since all substances in this reaction are in a gas phase, then, if the container volume is reduced to one fourth of its original value the equilibrium concentration of each component [A] increases 4 times.

$$K_{c_2} = \frac{[HI]^2}{[H_2]\times[I_2]} = \frac{4^2 \times [HI]^2}{5\times[H_2]\times4\times[I_2]}$$

$$= K_{C_1} = 64$$

54. 6

Sol.

	$PCl(g)$	$\rightleftharpoons$	$PCl_3(g)$	+	$Cl_2(g)$
At intial	4		0		0
At eqm.	$(4-4\alpha)$		4α		4α

where α is degree of dissociation

Total moles at equilibrium

$= 4 - 4\alpha + 4\alpha + 4\alpha$

$= 4 + 4\,\alpha = 4 + 4 \times 0.5 = 6$

55. 200

Sol. $C(s) + S_2(g) \rightleftharpoons CS_2(g)$

$$Pc = \frac{n}{v}Rt = \frac{1}{13}RT$$

$$P_{S_2} = \frac{1}{13}RT \text{ with } P_{CS_2} = \frac{1}{13}RT$$

$$n_c = \frac{12}{12} = 1\,\text{mole}$$

$$n_{S_2} = \frac{64}{64} = 1 \text{ mole}$$

$$n_{CS_2} = \frac{76}{76} = 1 \text{ mole}$$

Total pressure $= P_{C(g)} + P_{S_2(g)} + P_{CS_2(g)}$

$$= 0 + \frac{1}{13}RT + \frac{1}{13}RT$$

$$= \frac{1}{13} \times 2 \times R \times (1027 + 273) = 200R$$

Partial pressure of carbon in solid state

$P_{C(s)} = 0$ atm.

56. 1

Sol. Degree of dissociation $\alpha = 0.1 \times C^{-1}$

$$\begin{matrix} CH_3COOH & \rightleftharpoons & CH_3COO^- & + & H^+ \\ C - x & & x & & x \end{matrix}$$

$$\alpha = \frac{X}{c}$$

$$\Rightarrow \frac{0.1}{C} = \frac{x}{c} \Rightarrow x = \frac{1}{10}$$

$\therefore$ pH = 1

57. 0.16

Sol. $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$

$t = 0$	1.5	1.5	0
$t = t_{eq}$	$1.5 - x$	$1.5 - x$	$2x$

We know, $1.5 - x = 1.25$, or $x = 0.25$

$$K_c \frac{(0.5)^2}{(1.25)^2} = 0.16$$

58. 68

Sol. $$x = \frac{1.4.16 - 62}{62(2-1)} = 68\%$$

59. 9

Sol. According to given date [OH⁻] needed for precipitation of Mg(OH) $= \left(\frac{10^{-11}}{10^{-1}}\right)^{\frac{1}{2}}$

$= 10^{-5}$ M

pOH = log[OH–] = 5

So, pH = 14 – pOH = 9

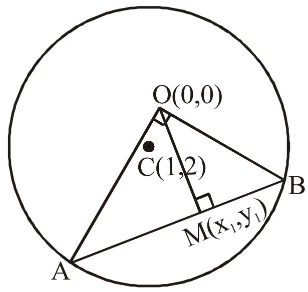
60. 5

Sol. $V \times 0.05 \times 2 = 0.5 \times 10 \times 0.1.$

V = 5 ml.

61. (A)

Sol. Equation to the chord AB is $(y - y_1) = \dfrac{-x_1}{y_1}(x - x_1)$



$\Rightarrow \quad xx_1 + yy_1 = x_1^2 + y_1^2 \quad$ ......(1)

Where M (x_1, y_1) is the foot of perpendicular from the origin.
Now homogenising the equation of the given circle, we get

$(x^2+y^2)(x_1^2+y_1^2)^2 - (2x+4y)(xx_1+yy_1)(x_1^2+y_1^2) - 4(xx_1+yy_1)^2 = 0$

This represents a pair of perpendicular lines passing through the origin.
Hence coefficient of x^2 + coefficient of $y^2 = 0$

$\Rightarrow \quad 2(x_1^2+y_1^2)^2 - (2x_1(x_1^2+y_1^2) + 4y_1(x_1^2+y_1^2)) - 4(x_1^2+y_1^2) = 0$

or $\quad (x_1^2+y_1^2) - (x_1+2y_1) - 2 = 0$

Hence locus of $M(x_1,y_1)$ is $x^2+y^2-x-2y-2=0$ **Ans.**]

62. (A)

Sol. Slope of AB $= \dfrac{7-1}{5-1} = \dfrac{3}{2}$

$\therefore$ slope of line perpendicular to AB $m = -\dfrac{3}{2}$

$\therefore$ equation $y - 4 = \dfrac{-2}{3}(x-3)$

$2x + 3y = 18$

63. (B)

Sol. Both sets of points are quite obviously Circles. To show this, we can rewrite each of them in the form $(x - x_0)^2 + (y - y_0)^2 = r^2$

The first curve becomes

$(x-6)^2 + (y-3)^2 = 7^2$

centre (6, 3) & r = 7

The second curve becomes

$(x-2)^2 + (y-6)^2 = 40 + k$

Centre (2, 6) & $r = \sqrt{40+k}$

The distance b/w two centre is 5 & therefore the two circles intersect if $2 \le r \le 12$

From $\sqrt{40+k} \ge 2$

$k \ge -36$

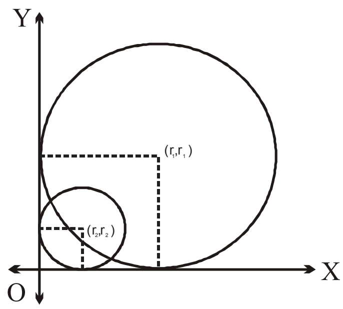
From $\sqrt{40+k} \le 12$

$k \le 104$

$\therefore \quad b - a = 104 - (-36) = 140$

64. (A)

Sol. Circle is $(x-r)^2 + (y-r)^2 = r^2$



$\Rightarrow \quad x^2 + y^2 - 2xr - 2yr + r^2 = 0$

Hence the circles are

$x^2 + y^2 - 2xr_1 - 2yr_1 + r_1^2 = 0 \quad$ ......(1)

$x^2 + y^2 - 2xr_2 - 2yr_2 + r_2^2 = 0 \quad$ .....(2)

As (1) and (2) are orthogonal so

$2r_1r_2 + 2r_1r_2 = r_1^2 + r_2^2$

$$4\frac{r_1}{r_2} = \left(\frac{r_1}{r_2}\right)^2 + 1 \Rightarrow \left(\frac{r_1}{r_2}\right)^2 - 4\left(\frac{r_1}{r_2}\right) + 1 = 0 \Rightarrow \frac{r_1}{r_2} = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm 2\sqrt{3}}{2}$$

$= 2 + \sqrt{3}$ or $2 - \sqrt{3}$ (rejected) **Ans.**]

65. (A)

Sol. $(x+5)^2+(y-12)^2=(14)^2$ ....(i)

Let $x^2+y^2=r^2$ ....(ii)

Distance b/w two circles

$C_1C_2=|r_1-r_2|$

$C_1=(-5,12),\ r_1=14$

$C_2=(0,0),\ r_2=r$

$$\sqrt{(-5-0)^2+(12-0)^2}=|14-r|$$

$=25+144=(14-r)^2$

$169=(14-r)^2$

$169=196+r^2 \quad -28r$

$x^2-28r+27=0$

$r^2-27r-r+27=0$

$r(r-27)-1\,(r-27)=0$

$(r-1)(r-27)=0$

$r=1$ or 27

$x^2+y^2=r^2$

$\therefore\ x^2+y^2=(1)^2$

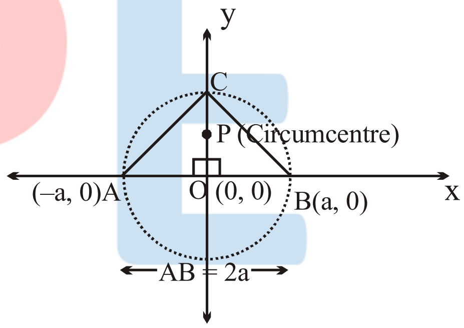
or

$x^2+y^2=(27)^2$

So, min. value of $x^2+y^2=1$

66. (C)

Sol. Let us assume 'c' as (x,y).



$CA^2=CB^2=AB^2$

$(x+a)^2+y^2=(x-a)^2+y^2=(2a)^r$

$\Rightarrow x^2+2ax+a^2+y^2=4a^2$..............(i)

$x^2-2ax+a^2+y^2=4a^2$..............(ii)

From (i) & (ii)

$x=0,\ y=\pm\sqrt{3}a$

Point C lies above x – axis & a > 0, So $y=\sqrt{3}a$

$\therefore$ C $(0,\ \sqrt{3}a\ )$

Let eq. of circumcircle be

$x^2+y^2+2gx+2fy+c=0$

Since point lies on circle

$\Rightarrow a^2-2ga+c=0$ ....(iii)

$a^2+2ga+c=0$ ....(iv)

$3a^2+2\sqrt{3}\ af+c=0$.........(v)

from (iii) (iv) & (v)

$$g=0,\ c=-a^2,\ f=\frac{-a}{\sqrt{3}}$$

equation of circumcircle is

$$x^2+y^2-\left(\frac{2a}{\sqrt{3}}\right)y-a^2=0$$

$$\Rightarrow x^2+y^2-\left(\frac{2\sqrt{3}ay}{3}\right)-a^2=0$$

$\Rightarrow 3x^2+3y^2-2\sqrt{3}\ ay=3\,a^2$

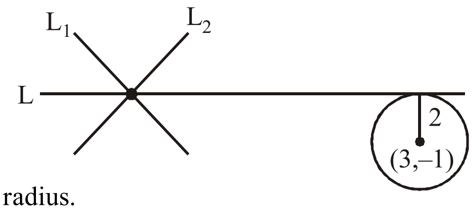
67. (D)

Sol. The line is $(x-2y-1)+\lambda(x+y-2)=0$ ....(1)

$\Rightarrow \quad x(1+\lambda)+y(\lambda-2)-(1+2\lambda)=0$

For the circle Centre $\equiv(3,-1)$

Radius $=\sqrt{9+1-6}=2$



$\because$ $\perp$ distance of line from the centre of circle is equal to radius.

$$\therefore \quad \frac{|3(1+\lambda)-(\lambda-2)-(1+2\lambda)|}{\sqrt{(1+\lambda)^2+(\lambda-2)^2}}=2$$

$\Rightarrow \quad (3+3\lambda-\lambda+2-1-2\lambda)^2 = 4[1+\lambda^2+2\lambda+\lambda^2-4\lambda+4]$

$\Rightarrow \quad 4^2=4(2\lambda^2-2\lambda+5) \quad \Rightarrow \quad 2\lambda^2-2\lambda+1=0$

$\because \quad D<0 \Rightarrow$ No real λ

$\therefore$ No such line exists

68. (A)

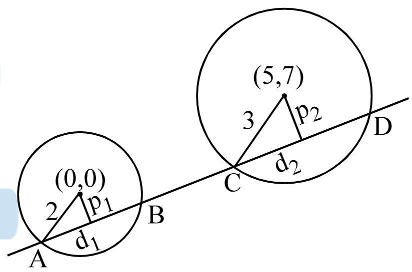
Sol. Let equation of line be $y=x+c$

$y-x=c$ ....(1) [Note that $d_1=d_2$]

perpendicular from (0, 0) on (1) is $\left|\frac{-c}{\sqrt{2}}\right|=\frac{c}{\sqrt{2}}$

In ΔAON, $\sqrt{2^2-\left(\frac{c}{\sqrt{2}}\right)^2}=AN$

and in ΔCPM, $\sqrt{3^2-\left(2-\frac{c}{\sqrt{2}}\right)}=CM$



perpendicular from (5, 7) on line $y-x=c=\frac{2-c}{\sqrt{2}}$

Given $AN=CM=4-\frac{c^2}{2}=9-\frac{(2-c)^2}{2} \Rightarrow c=-\frac{3}{2}$

$\therefore$ equation of line $y=x-\frac{3}{2}$ of $2x-2y-3=0$]

69. (D)

Sol. Given equation is

$x^2+2x(y+g)+y^2+2fy+4=0$

$$2x=-2(y+g)\pm\sqrt{4(y+g)^2-4(y^2+2fy+4)}$$

$x=-(y+g)\pm\sqrt{(g^2-4)+2y(g-f)}$ ....(1)

(1) will represent a pair of lines if its discriminant is zero

$\Rightarrow \quad +4(g-f)=0 \quad \Rightarrow \quad g=f$

$\therefore \quad x=-(y+g)\pm\sqrt{g^2-4}$. For two real lines

$g^2\geq 4 \quad \Rightarrow \quad g\geq 2$ or $g\leq -2$ **Ans.**]

70. (D)

Sol. Equation of the ray:

$y - 1 = m(x - 2)$

$mx - y - 2m + 1 = 0$

$\because$ touches the circle

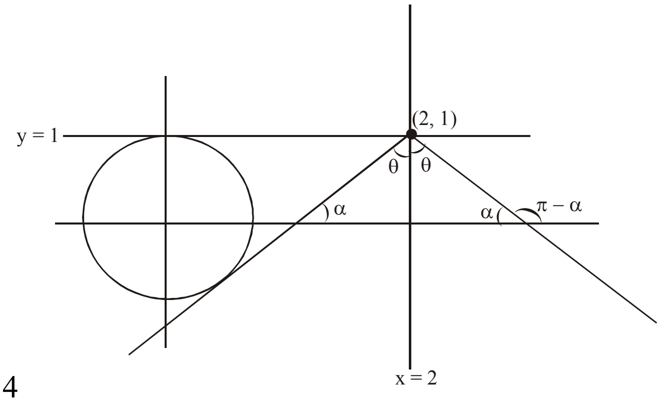
$\therefore \quad p = r$

$\left|\dfrac{1-2m}{\sqrt{1+m^2}}\right| = 1$

$1 + 4m^2 - 4m = 1 + m^2$

$\because \quad m \neq 0$

$4m - 4 = m$



$m = \dfrac{4}{3}$ $\qquad \tan\alpha = \dfrac{4}{3}$

$\tan(\pi - \alpha) = -\dfrac{4}{3}$

$\therefore$ Equation of incident ray

$y - 1 = -\dfrac{4}{3}(x - 2)$

$3y - 3 = -4x + 8$

$4x + 3y = 11$ Ans.]

71. (D)

Sol. Let the equation of line parallel to

$3x - y - 1 = 0$

$3x - y + k = 0$ …(1)

it passes through (1, 2)

$\therefore 3 - 2 + k = 0 \quad \Rightarrow k = -1$

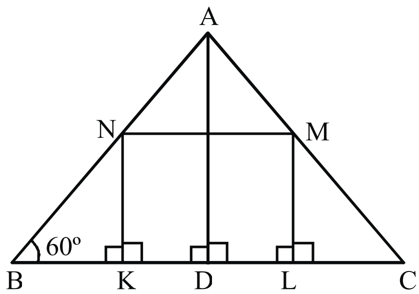
$\therefore$ equation $3x - y - 1 = 0$

72. (D)

Sol. Here, $\dfrac{AN}{2} = \dfrac{NB}{1} = \dfrac{AB}{3}$

$\therefore \dfrac{NB}{AB} = \dfrac{1}{3}$ ……(1)

As, $\Delta BKN \sim \Delta BDA$ (A – A)



So, $\dfrac{ar.(\Delta BKN)}{ar.(\Delta BDA)} = \dfrac{(BN)^2}{(AB)^2} = \dfrac{1}{9}$

$\Rightarrow ar.(\Delta BDA) = 54$

$\therefore ar.(\Delta ABC) = 2 \times \Delta BDA = 2 \times 54 = 108$

73. (C)

Sol. (x_1, y_2) lies on line

$\therefore 3x_1 - 4y_2 - a(a-2) = 0$

$\therefore 4y_2 = 3x_1 - a(a-2) = 0$

Now, $y_2 < y_1 \Rightarrow \dfrac{3x_1 - a(a-2)}{4} < y_1$

$\Rightarrow 3\,(2b + 3) - a\,(a - 2) < 4b^2$

$\Rightarrow a^2 - 2a + 4b^2 - 6b - 9 > 0 \ \forall\ a \in R$

$\Rightarrow D < 0 \Rightarrow 4 - 4\,(4b^2 - 6b - 9) < 0$

$\Rightarrow 1 - 4b^2 + 6b + 9 < 0$

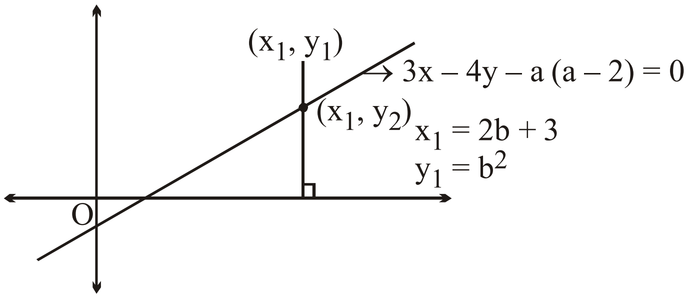
$\Rightarrow 4b^2 - 6b - 10 > 0$

$\Rightarrow 2b^2 - 3b - 5 > 0$

$\Rightarrow (2b - 5)\,(b + 1) > 0$

$\Rightarrow b \in (-\infty, -1) \cup \left(\frac{5}{2}, \infty\right)$

Put $x_1 = 2b + 3;\ y_1 = b^2$



Hence, least positive integral value 'b' is 3.

74. (D)

Sol. $x^2 - y^2 + 2x + 4y - k = 0$

The above equation is a pair of straight lines

$\therefore\ k = 3.$

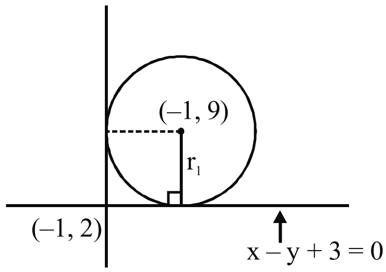
$x^2 - y^2 + 2x + 4y - 3 = 0$

$\Rightarrow\ x - y + 3 = 0$ and $x + y - 1 = 0$

$$|r| = \left|\frac{-1 - 9 + 3}{\sqrt{2}}\right| = \frac{7}{\sqrt{2}}$$

$2r^2 = 49$

$\therefore\ k + 2r^2 = 52$ **Ans.**]

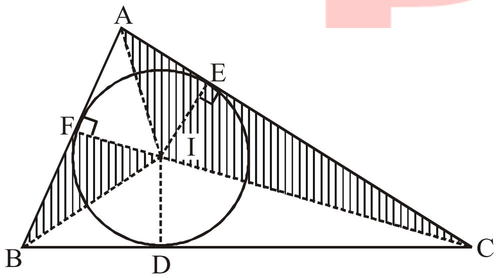


75. (B)

Sol. ΔIEC and ΔICD are congruent

|||ly ΔIAE and ΔIAF are congruent and ΔIBD and ΔIBF are congruent.

Hence Area (ΔABC) = 2 (Area (ΔIBF) + Area (ΔIAC)) = 2(2 + 4) = 12 sq. units



76. (A)

Sol. The point of intersection the lines $x - 2y - 3 = 0$ and $3x - 2y - 5 = 0$ is B (1,–1). BP is the normal at P. Clearly, BP passes through B (1 –1) and perpendicular to $3x - 2y - 5 = 0$. So, equation of BP is

$y + 1 = -(2/3)(x - 1)$ or, $2x + 3y + 1 = 0$

Since the reflected ray passes through the intersection of $x - 2y - 3 = 0$ and the normal $2x + 3y + 1 = 0$ Therefore, equation of the reflected ray is

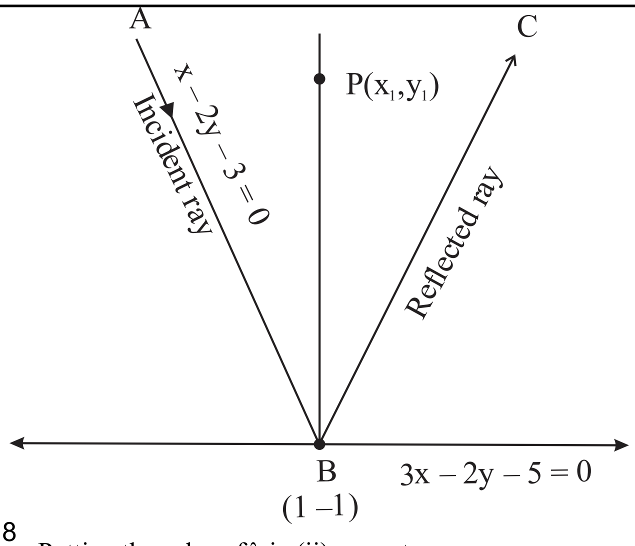
$x - 2y - 3 + \lambda(2x + 3y + 1) = 0$ ....(i)

or $x(1 + 2\lambda) + y(3\lambda - 2) + (\lambda - 3) = 0$ ....(ii)

Let $P(x_1, y_1)$ be an arbitrary point on the normal at P Then, P is equidistant from the incident ray and the Reflected ray.

$$\left|\frac{x_1 - 2y_1 - 3}{\sqrt{1+4}}\right| = \left|\frac{(x_1 - 2y_1 - 3) + \lambda(2x_1 + 3y_1 + 1)}{\sqrt{(1+2\lambda)^2 + (3\lambda - 2)^2}}\right|$$

$$\Rightarrow \left|\frac{x_1 - 2y - 3}{\sqrt{5}}\right| = \left|\frac{(x_1 - 2y_1 - 3) + \lambda \times 0}{\sqrt{(1+2\lambda)^2 + (3\lambda - 2)^2}}\right| \quad \left[\begin{matrix}\because (x_1, y_1) \text{ lies on } 2x + 3y + 1 = 0 \\ 2x_1 + 3y_1 + 1 = 0\end{matrix}\right]$$



$\Rightarrow \quad \left|\frac{1}{\sqrt{5}}\right| = \left|\frac{1}{\sqrt{(2\lambda+1)^2+(3\lambda-2)^2}}\right|$

$\Rightarrow \quad 5 = (2\lambda+1)^2 + (3\lambda-2)^2$

$\Rightarrow \quad 13\lambda^2 - 8\lambda = 0 \Rightarrow \lambda = 0 \text{ or}, \lambda = \frac{8}{13}$

Since $\lambda = 0$ is not possible. So, $\lambda = \frac{8}{13}$ Putting the value of λ in (ii), we get

$29x - 2y - 31 = 0$ as the equation of the line containing the reflected ray.

77. (A)

Sol. $3x + 4y = 9 \,;\, y = mx + 1$

$3x + 4(mx + 1) = 9$

$(3 + 4m)\,x = 9 - 4 = 5 \qquad \Rightarrow x = \frac{5}{3+4m}$

x will be are integer if

$3 + 4m$ is a divisor of 5

$\Rightarrow \quad 3 + 4m = 1$ or -1 or 5 or -5

$3 + 4m = 1 \Rightarrow \quad m = \frac{1}{2}$ (Rejected)

$3 + 4m = -1$

$m = -1$

$3 + 4m = 5$

$m = -\frac{1}{2}$ (Rejected)

$3 + 4m = -5$

$m = -2.$

Hence $\quad m \in \{-1, -2\} \Rightarrow$ A.]

78. (A)

Sol. $x^2 - y^2 + 2y = 1$

$x^2 = (y - 1)^2$

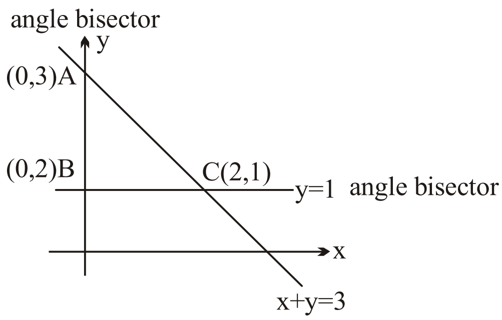
$x = y - 1$ or $\qquad x = -y + 1$

$x - y + 1 = 0 \qquad$ or $\qquad -x - y + 1 = 0$

$\therefore \quad$ angle bisectors are

$$\frac{x-y+1}{\sqrt{2}} = \pm\frac{(-x-y+1)}{\sqrt{2}}$$

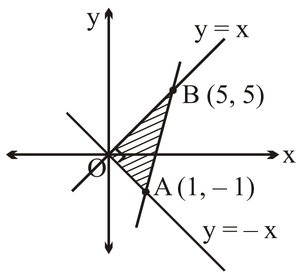
(+)	(–)
$2x = 0$	$2y = 2$
$x = 0$	$y = 1$



$\therefore \quad$ Area of $\Delta ABC = \frac{1}{2} \times 2 \times 2 = 2$]

79. (D)

Sol. Equation of line through (3, 2) with slope $\frac{3}{2}$ is $(y-2) = \frac{3}{2}(x-3)$



$\Rightarrow 3x - 2y = 5$

Here, O (0, 0), A (1, – 1), B (5, 5)

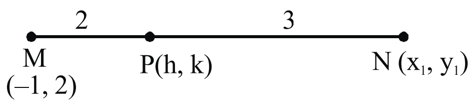
Now, OA = $\sqrt{2}$, OB = $5\sqrt{2}$

$\Rightarrow$ Area of ΔOAB = $\frac{1}{2}\left(\sqrt{2}\right)\left(5\sqrt{2}\right) = 5$. **Ans.**]

80. (A)

Sol. Let $N(x_1, y_1)$ and N lies on $xy = 4$

$x_1y_1 = 4$ ……(1)



$h = \frac{2x_1 + (-3)}{5}$ and $k = \frac{2y_1 + 6}{5}$

$x_1 = \frac{5h+3}{2}$ and $y_1 = \frac{5k-6}{2}$

Put x_1 and y_1 in (1)

$\left(\frac{5h+3}{2}\right)\left(\frac{5k-6}{2}\right) = 4$

$25hk - 30h + 15k - 18 = 16$

$25xy - 30x + 15y = 34$]

81. 4

Sol. Slope of line passes through (4, 3) and (2, k)

$m_1 = \frac{k-3}{2-4} = \frac{k-3}{-2}$

slope of line $y = 2x + 3$ is $m_2 = 2$

lines are perpendicular

$\therefore m_1m_2 = -1$

$\Rightarrow \left(\frac{k-3}{-2}\right)(B) = -1 \Rightarrow k = 4$

82. 2

Sol. The perpendicular distance of (1, 3) from the line $3x + 4y = 5$ is 2 units while,

$\sec^2\theta + 2\,\text{cosec}^2\theta \geq 3$ {as $\sec^2\theta, \text{cosec}^2\theta \geq 1$}

Evidently, these will be two such points on the line.

83. 64

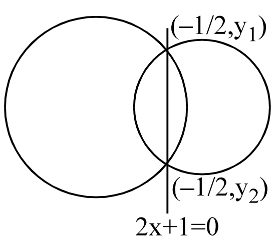
Sol. **K:** Radical axis of the two circles is,

$2x + 1 = 0$

hence $k = (AB)^2 = (y_1 - y_2)^2 = (y_1 + y_2)^2 - 4y_1y_2$

now solving $2x + 1 = 0$ and

$x^2 + y^2 + 2x + 3y + 1 = 0$



$\frac{1}{4} + y^2 - 1 + 3y + 1 = 0$

$4y^2 + 12y + 1 = 0$

$y_1 + y_2 = -3$; $y_1y_2 = \frac{1}{4}$; $k = |y_1 - y_2|^2$

hence $K = (y_1 + y_2)^2 - 4y_1y_2 = 9 - 1 = 8$

W: Let the variable chord of $y^2 = 8x$ be, $y = mx + c$

or $\dfrac{y - mx}{c} = 1$ ....(1)

Homogenizing $y^2 = 8x$

with the help of (1), we get

$cy^2 = 8x(y - mx) \Rightarrow 8mx^2 + cy^2 - 8xy = 0$

now coefficient of x^2 + coefficient of $y^2 = 0$

$8m + c = 0 \Rightarrow c = -8m$

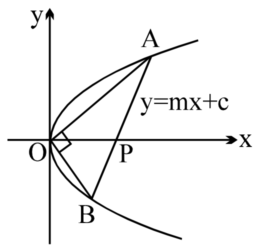
hence equation of the variable chord is

$y = mx - 8m$

$y - 0 = m(x - 8)$

hence point of concurrence is (8, 0)

$\Rightarrow W = 8$



H: Circle is $x^2 + y^2 + \dfrac{5}{2}y - 8 = 0$ $\{p = (3, 0)\}$

$\therefore$ $H = L^2 = 9 - 8 = 1$

hence $KWH = 8 \cdot 8 \cdot 1 = 64$ **Ans.**

84. 38

Sol. Line $S_1 - S_2 = 0$ i.e. $4x + 14y + (a + b) = 0$

passes through center of $S_2 = 0$

[Centre of $S_2 = (-1, 3)$]

$\Rightarrow -4 + 42 + (a + b) = 0$

$\Rightarrow a + b = -38$

85. 40

Sol. The equation of circle taking AB as diameter

$(x - 2)(x - 4) + (y - 1)(y - 3) = 0$ ....(1)

The equation of the line joining the points A and B is

$x - y - 1 = 0$ ....(2)

The equation of members of family of circle passing through A and B is given by

$S \equiv (x - 2)(x - 4) + (y - 1)(y - 3) + \lambda(x - y - 1) = 0$ where λ is parameter, $\lambda \in R$

$\therefore$ $S \equiv x^2 + y^2 + (\lambda - 6)x + (-\lambda - 4)y + (11 - \lambda) = 0$ ....(3)

Let the circle which cuts the members of circle be

$S_1 = x^2 + y^2 + 2gx + 2fy + c = 0$ ....(4)

Applying condition of orthogonality for (1) and (2), we get

$$2g\left(\frac{\lambda - 6}{2}\right) + 2f\left(\frac{-\lambda - 4}{2}\right) = c + 11 - \lambda$$

i.e. $(-6g - 4f - c - 11) + \lambda(g - f + 1) = 0$

This will also hold for all $\lambda \in R$

$\therefore$ we have $-6g - 4f - c - 11 = 0$ and $g - f + 1 = 0$

solving these equations for g and f in terms of c, we get $g = \dfrac{-c - 15}{10}$ and $f = \dfrac{-c - 5}{10}$

substituting the values of g and f in terms of c in (2), we get the circles cutting the circles of system (4) orthogonally as

$$x^2 + y^2 + 2\left(\frac{-c - 15}{10}\right)x + 2\left(\frac{-c - 5}{10}\right)y + c = 0$$

or $x^2 + y^2 - 3x - y - \dfrac{c}{5}(x + y - 5) = 0$

which represents equation of family of circles passing through two fixed points and cutting the family of circles orthogonally whose coordinates obtained by solving equations

i.e. solving $x^2+y^2-3x-y=0$ and $x+y-5=0$

$\Rightarrow$ $x^2-6x-10=0$ $(D<0)$

$x_1+x_2=6$; $x_1x_2=10$

|||ly $y^2-4y+5=0$

$y_1+y_2=4$; $y_1y_2=5$

$$\left(x_1^3+x_2^3+y_1^3+y_2^3\right)=(x_1+x_2)^3-3x_1x_2(x_1+x_2)+(y_1+y_2)^3-3y_1y_2(y_1+y_2)$$

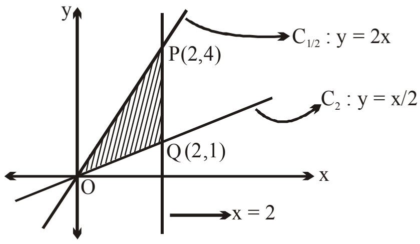
$$=216-30(6)+64-60$$

$$=36+4=40$$ **Ans.**]

86. 3

Sol. Let midpoint of A and B be (h, k). So equation of chord to the curve $y^2-x^2=0$, is $(T=S_1)$

i.e., $ky-hx=k^2-h^2$



Clearly, slope $=\dfrac{h}{k}=\beta$ (Given)

$\therefore$ Locus of (h, k) is

$x=\beta y\Rightarrow c_\beta\equiv x-\beta y=0.$

$c_2: x-2y=0$

$c_{1/2}: 2x-y=0$

and $x=2.$

Area enclosed by the curves is

$$\text{Ar}(\Delta OPQ)=\frac{1}{2}\times 3\times 2=3 \text{ sq. units.}$$ **Ans.**]

87. 9

Sol. Clearly, the length of the side of the square is equal to the distance between the parallel lines

$x+y-1=0$ ....(i) and $x+y+2=0$ ......(ii)

Putting $x=0$ in (i), we get $y=1$. So (0,1) is a point on line (i).

Distance between the parallel lines

$$=\{\text{Length of the Perpendicular from } (0,1) \text{ to } x+y+2=0\}=\frac{|0+1+2|}{\sqrt{1^2+1^2}}=\frac{3}{\sqrt{2}}$$

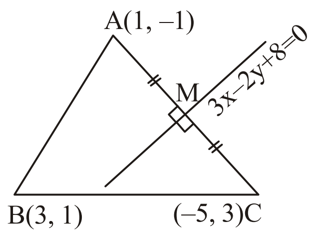
Thus, the length of the side of the square is $\dfrac{3}{\sqrt{2}}$ and hence its area is $\left(\dfrac{3}{\sqrt{2}}\right)^2=\dfrac{9}{2}$ square units

Since, $k=\dfrac{9}{2}$

therefore, $2k=2\times\dfrac{9}{2}=9$ square units

88. 11

Sol. Slope of line AC $=\dfrac{-1}{\frac{-3}{-2}}=\dfrac{-2}{3}$



Equation of line AC is $(y+1)=\dfrac{-2}{3}(x-1)$

$\Rightarrow$ $3y+3=-2x+2$ $\Rightarrow$ $2x+3y=-1$

$\Rightarrow$ $6x+9y=-3$ ... (1)

Equation of line perpendicular is $6x-4y=-16$ ...(2)

$\therefore$ On solving (1) and (2), we get M $(-2, 1)$

For $C(x_1, y_1)$, we have

$$\frac{x_1+1}{2} = -2 \quad \Rightarrow \quad x_1 = -4-1 = -5$$

$$\frac{y_1-1}{2} = 1 \quad \Rightarrow \quad y_1 = 2+1 = 3$$

$\therefore$ C (– 5, 3) and B (3, 1)

Equation of BC is

$$(y-1) = \frac{3-1}{-5-3}(x-3) \quad \Rightarrow \ -8(y-1) = 2(x-3) \ \Rightarrow \ -4(y-1) = x-3$$

$\Rightarrow \quad -4y+4 = x-3 \quad \Rightarrow \quad x+4y = 7 \ \Rightarrow \ a = 4$ and $b = 7 \ \Rightarrow \ a+b = 11$ **Ans.**]

89. 11

Sol. The lines are

$$\left.\begin{aligned} y &= \frac{-1}{3}x + \frac{a}{3} \\ y &= \frac{-1}{3}x - \frac{4a}{3} \end{aligned}\right] \quad \text{and} \quad \begin{aligned} y &= \frac{3x}{2} + \frac{3a}{2} \\ y &= \frac{3x}{2} + \frac{7a}{2} \end{aligned}$$

Now $\quad A = \dfrac{|c_1 - c_2||d_1 - d_2|}{|m_1 - m_2|}$

Hence $\quad A = \dfrac{\left(\frac{5a}{3}\right)2a \cdot 6}{11}$

$$220 = \frac{20a^2}{11} \ \Rightarrow \ a^2 = 121 \ \Rightarrow \ a = 11 \ \textbf{Ans.]}$$

90. 8

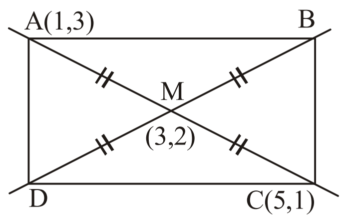
Sol. Let the equation of BD with slope is

$y = 2x + c \qquad$ ....(1)

middle point of AC i.e. (3, 2) lies on (1)

hence $\quad 2 = 6 + c \quad \Rightarrow \quad c = -4$

equation of BD is $\ y = 2x - 4$



now $\quad AC = \sqrt{16+4} = 2\sqrt{5}$

hence $\quad BD = 2\sqrt{5}$

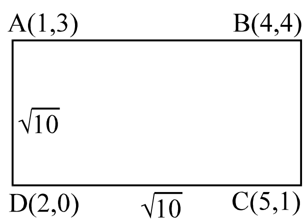
$\therefore \quad MD = MB = \sqrt{5}$

taking parametric length M, equation of DB is

$$\frac{x-3}{\cos\theta} = \frac{y-2}{\sin\theta} = \pm\sqrt{5}$$

$x = 3+\sqrt{5}\cos\theta \quad$ or $\quad x = 3-\sqrt{5}\cos\theta$

$y = 2+\sqrt{5}\sin\theta \quad$ or $\quad y = 2-\sqrt{5}\sin\theta$



where $\tan\theta = 2 \quad \Rightarrow \quad \cos\theta = \dfrac{1}{\sqrt{5}}$ and $\sin\theta = \dfrac{2}{\sqrt{5}}$

hence $\quad x = 4; \quad y = 4 \quad$ or $\quad x = 2; \ y = 0$

Hence $\quad$ B is (4, 4) and D is (2, 0)

product of abscissae of B and D is $4 \times 2 = 8$. **Ans.**]