JEE MAIN ANSWER KEY & SOLUTION

PAPER CODE :- PART TEST-4
CLASS-XII

ANSWER KEY

PHYSICS

1.	(B)	**2.**	(C)	**3.**	(A)	**4.**	(A)	**5.**	(B)	**6.**	(B)	**7.**	(C)
8.	(C)	**9.**	(B)	**10.**	(C)	**11.**	(B)	**12.**	(D)	**13.**	(D)	**14.**	(A)
15.	(B)	**16.**	(B)	**17.**	(A)	**18.**	(A)	**19.**	(D)	**20.**	(A)	**21.**	3
22.	0	**23.**	2	**24.**	12	**25.**	5	**26.**	18	**27.**	80	**28.**	49
29.	48	**30.**	30										

CHEMISTRY

31.	(A)	**32.**	(D)	**33.**	(A)	**34.**	(C)	**35.**	(C)	**36.**	(D)	**37.**	(C)
38.	(D)	**39.**	(A)	**40.**	(C)	**41.**	(B)	**42.**	(A)	**43.**	(B)	**44.**	(C)
45.	(C)	**46.**	(C)	**47.**	(A)	**48.**	(A)	**49.**	(C)	**50.**	(B)	**51.**	4
52.	4	**53.**	2	**54.**	3	**55.**	2	**56.**	5	**57.**	4	**58.**	3
59.	6	**60.**	5										

MATHEMATICS

61.	(D)	**62.**	(A)	**63.**	(A)	**64.**	(A)	**65.**	(C)	**66.**	(B)	**67.**	(B)
68.	(C)	**69.**	(B)	**70.**	(B)	**71.**	(B)	**72.**	(A)	**73.**	(B)	**74.**	(C)
75.	(B)	**76.**	(D)	**77.**	(A)	**78.**	(A)	**79.**	(B)	**80.**	(B)	**81.**	82
82.	375	**83.**	2	**84.**	2	**85.**	2	**86.**	48	**87.**	40	**88.**	79
89.	3	**90.**	3										

PHYSICS

1. (B)

Sol. $I_0 \left| \overset{A}{\frac{I_0}{2}} \right| \left| \overset{C}{\frac{3I_0}{8}} \right| \overset{B}{\frac{3I_0}{32}}$

$I_C = \frac{I_0}{2}\cos^2 30^\circ = \frac{3I_0}{8}$

$I_B = \frac{3I_0}{8}\cos^2 60^\circ = \frac{3I_0}{32}$

$\% = \frac{3I_0 \times 100}{32 I_0} = 9.4\,\%$

2. (C)

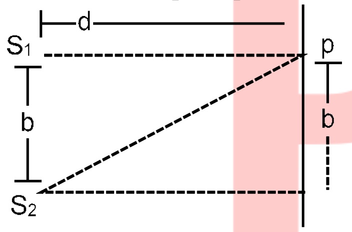
Sol. $\frac{1}{10} = \left(\frac{3}{2} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ ....(1)

when placed in medium of RI $\frac{7}{5}\frac{1}{F'} = \left(\frac{3/2}{7/5} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ ....(2)

Dividing (1) by (2) $F' = 70$ cm

3. (A)

Sol. If minima is observed at point p



Path difference $= \sqrt{b^2 + d^2} - d$

$d\left[1 + \frac{b^2}{d^2}\right]^{\frac{1}{2}} - d \quad \Rightarrow \quad d + \frac{b^2}{2d} - d$

If $= \frac{b^2}{2d} = \frac{\lambda}{2}$

$\lambda = \frac{b^2}{d}$

4. (A)

5. (B)

Sol. $K = 4I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$

$\Delta\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$

$K' = 4I_0 \cos^2\left(\frac{\pi}{4}\right)$

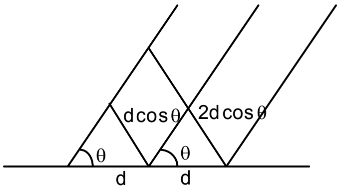
$= 2I_0 = \frac{K}{2}$

6. (B)

Sol. Image by convex mirror is always virtual, erect and diminised.
In case of concave mirror, see using position of object.

7. (C)

Sol.



∴ for minima

$d \cos\theta = \lambda/3$

$\cos\theta = \lambda/3d = 1/9$

$$\tan\theta = 4\sqrt{5} = \frac{x}{D}$$

$$x = 4\sqrt{5}\,D$$

8. (C)

Sol. Thickness of glass slab t = 15 cm. Apparent depth through one face = 6 cm, apparent depth through opposite face = 4cm.
Let x1 and x2 be the real distance of the bubble from two faces. The refractive index of the glass is

$$\mu_g = \frac{x_1}{6} \quad \text{or} \quad x_1 = 6\mu_g \qquad \text{... (1)}$$

Similarly the refractive index when viewed from opposite face is given by

$$\mu_g = \frac{x_2}{4} \quad \text{or} \quad x_2 = 4\mu_g \qquad \text{... (2)}$$

Hence, the thickness of the slab is given by
t = x1 + x2 [from (1) and (2)]

$$15 = 6\mu_g + 4\mu_g = 10\mu_g$$

or $\mu_g = \frac{15}{10} = 1.5$

9. (B)

Sol. $(\mu - 1)d = d\sin\theta$

$\sin\theta = (\mu - 1)$

$\sin\theta = 30^\circ$

$$\tan\theta = \frac{y}{D}$$

$$y = \frac{D}{\sqrt{3}}.$$

10. (C)

Sol. As $\frac{dV}{dt} = -\frac{V^2}{u^2}\frac{du}{dt}$

Now, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$\frac{1}{-12} = \frac{-1}{20} + \frac{1}{v} \Rightarrow v = -30 \text{ cm}$$

∴ $\frac{dv}{dt} = -9$ cm/s

i.e., 9, away from the mirror.

11. (B)

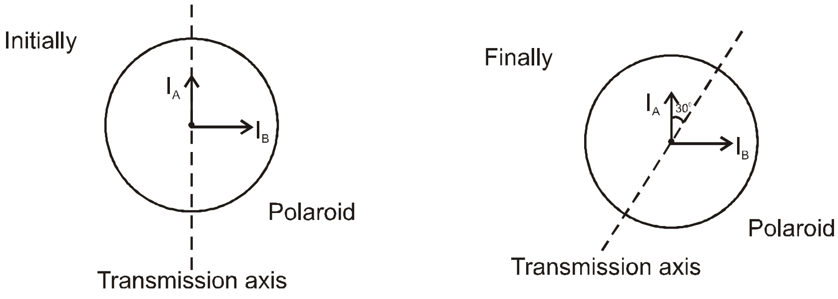
12. (D)

Sol. Put $A = \delta min$ and $\mu = \sqrt{2}$

The relation $\mu = \dfrac{\sin\left(\dfrac{A+\delta_{min}}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$ and solve for A

13. (D)

Sol.

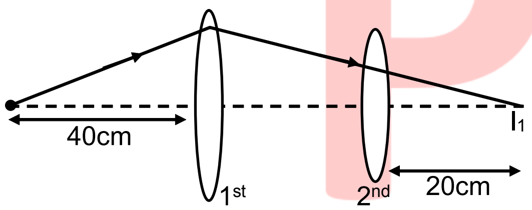


$I_A \cos^2 30^0 = I_B \cos^2 60^0$

$\Rightarrow I_A \times \dfrac{3}{4} = I_B \times \dfrac{1}{4} \quad \Rightarrow \quad \dfrac{I_A}{I_B} = \dfrac{1}{3}$

14. (A)

Sol.



$\dfrac{1}{v} - \dfrac{1}{-40} = \dfrac{1}{20} \quad \Rightarrow \quad -\dfrac{1}{v} = \dfrac{1}{20} - \dfrac{1}{40} \quad \Rightarrow \quad v = 40 \text{ cm}$

for 2nd lens

$-\dfrac{1}{v} = \dfrac{1}{u} = \dfrac{1}{f} \quad \Rightarrow \quad \dfrac{1}{v} - \dfrac{1}{20} = \dfrac{1}{20} \quad \Rightarrow \quad v = 10 \text{ cm}$

$m = \dfrac{10}{20} = \dfrac{1}{2}$

$-\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

$-\dfrac{1}{v^2} - \dfrac{dv}{dt} + \dfrac{1}{u^2}\dfrac{du}{dt} = 0$

$\dfrac{dv}{dt} = \dfrac{v^2}{u^2}\dfrac{du}{dt}$

$\vec{v}_{IL} = m^2 \vec{v}_{OL}$

$\vec{v}_I - 7\hat{i} = -\dfrac{7}{4}\hat{i}$

$\vec{v}_I = \dfrac{21}{4}\hat{i}$

$k = 3$

15. (B)

Sol. $\theta = \dfrac{\lambda}{a}$

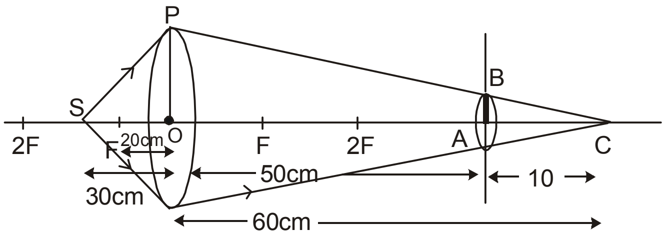
$\lambda_v < \lambda_R$

$\theta_v < \theta_R$

16. (B)

Sol. $\dfrac{1}{f} = -\dfrac{1}{u} + \dfrac{1}{v}$

$\dfrac{1}{20} = \dfrac{+1}{+30} + \dfrac{1}{v}$



$\dfrac{1}{v} = \dfrac{1}{20} - \dfrac{1}{30} = \dfrac{1}{60}$

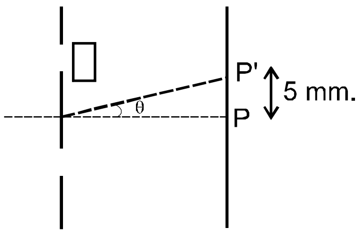
v = 60 cm

As triangles OPC & ABC are similar

$\therefore \quad \dfrac{2}{60} = \dfrac{x}{10} \quad \Rightarrow \quad x = \dfrac{1}{3} = 0.3$ cm.

17. (A)

Sol.



Clearly the central maxima at P(initially) shifts to P' where PP' = 5 mm.

So now, path difference at P' must be zero.

$\Rightarrow \quad d\sin\theta = (\mu - 1)t$

$\Rightarrow \quad d\tan\theta = (\mu - 1)t$

$\mu = 1 + \dfrac{d.(PP')}{Dt}$; get $\quad \mu = 1.2$

18. (A)

Sol. In a prism : r + r' = A

$\Rightarrow \quad r = A - r'$

$\therefore \quad r = 60^o - (10 + t2) = 50 - t2$

19. (D)

Sol. $I_R = I_0 \cos^2 \frac{\phi}{2}$

$\phi = \frac{2\pi}{\lambda}(\Delta x) = \frac{\pi}{3}$

$\because I_R = \cos^2 \frac{\pi}{6}$

$\frac{I}{I_0} = \frac{3}{4}$

20. (A)

Sol. From Newton's equation of lens

size of object = O2 = I1I2

where I1 is size of Image of object and I2 is size of image when pisitions of object & image are interchanged

So A2 = A1A2 $\Rightarrow$ A = $\sqrt{A_1A_2}$

21. 3

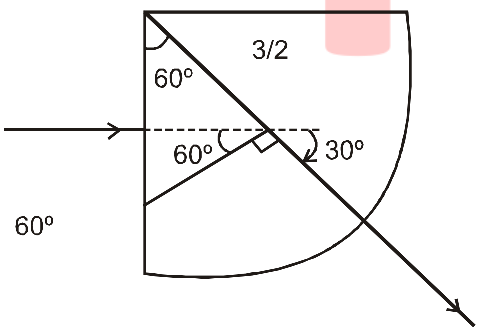
Sol. $\frac{1}{x} - \frac{1}{-x} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{\infty} - \frac{1}{-300}\right) \Rightarrow \frac{2}{x} = \frac{1}{2} \cdot \frac{1}{300} \Rightarrow$ x = 1200 cm = 12m

so, 3m

22. 0

23. 2

Sol.



$\sqrt{3}$ sin 60 = 1.5 sin r

$\Rightarrow$ r = 90°

δ = 30°

24. 12

Sol. When coherent then Δx at centre = 0

$\therefore I_{net} = 4I$

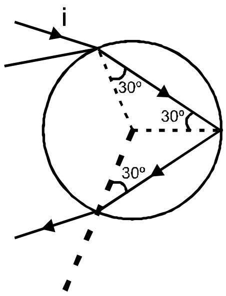
when Incoherent

$I'_{net} = I + I = 2I$

Ratio = $\frac{I_{net}}{I'_{net}} = \frac{2}{1} = 2$

25. 5

Sol. $\frac{\sin i}{\sin r} = \sqrt{2} \quad \Rightarrow \quad i = 45^\circ$



Total deviation

$= (45^\circ - 30^\circ) + 180^\circ - 2(30^\circ) + (45^\circ - 30^\circ)$

$= 30^\circ + 120^\circ$

$= 150^\circ = 30x$

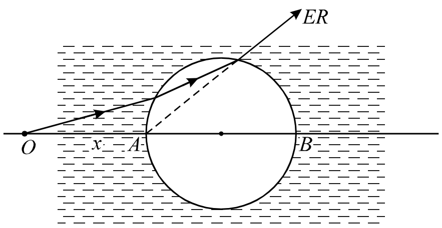
$x = 5.$

26. 18

Sol. Fringe width, $w = \frac{\lambda D}{d} \propto \lambda$ wavelength is decreasing from 600 nm to 400 nm so fringe width is also decreasing by a factor of $\frac{4}{6}$ or $\frac{2}{3}$. So no. of fringes will increase by a factor of $\frac{3}{2}$.

27. 80

Sol. For the given case Ray diagram will be as given here



Here say ER will be seen by observer which appear to becoming from point A. To find x, the distance of object from A we reverse the light rays by considering ER as incident ray & find the position of image after two refractions.

For I refraction we use

$$\frac{1}{v} - \frac{4/3}{2R} = \frac{1 - 4/3}{-R} \quad \Rightarrow \quad v = -3R$$

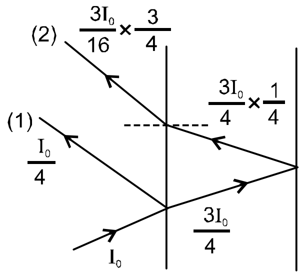
For II refraction we use

$$\frac{4/3}{x} - \frac{1}{-R} = \frac{4/3 - 1}{R} \quad \Rightarrow \quad x = -2R$$

Hence OA $= 2 \times 4 = 8$ cm.

28. 49

Sol. The shown diagram represents the reflection and refraction intensity.

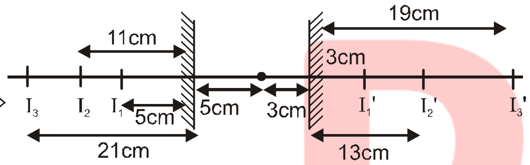


Hence the 2 waves interfering, 1 and 2, have intensities $\frac{I_0}{4}$ and $\frac{9I_0}{64}$.

$$\Rightarrow \frac{I_{min}}{I_{max}} = \frac{\left(\sqrt{\frac{I_0}{4}} - \sqrt{\frac{9\ \ I_0}{64}}\right)^2}{\left(\sqrt{\frac{I_0}{4}} + \sqrt{\frac{9I_0}{64}}\right)^2} = \frac{\left(\frac{1}{2} - \frac{3}{8}\right)^2}{\left(\frac{1}{2} + \frac{3}{8}\right)^2} = \frac{1}{49}$$

29. 48

Sol. The positions of images are shown

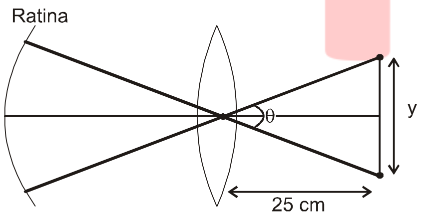
$\Rightarrow$ 

Required distance (21 + 8 + 19)cm = 48 cm

Ans. **48**

30. 30

Sol.



Resolving angle of necked eye is given by :

$$\theta = 1.22\frac{\lambda}{D}$$

$$\frac{y}{25 \times 10^{-2}} = \frac{1.22 \times 500 \times 10^{-9}}{0.25 \times 2 \times 10^{-2}}$$

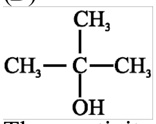
$y = 30 \times 10^{-6}$ m = 30 μm

31. (A)

Sol. $R-CH_2-X$ is primary alkyl halide

32. (D)

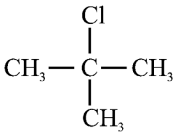
Sol.



The reactivity of lucas reagent : 3° > 2° > 1°

33. (A)

Sol.



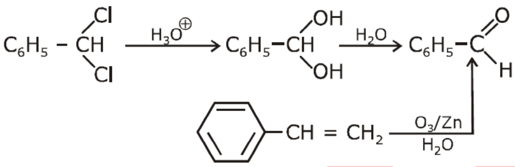
,2-chloro-2-methyl propane.

34. (C)

Sol. Allyl chloride

35. (C)

Sol.



36. (D)

Sol. None of these

37. (C)

Sol. Dipole – Dipole attraction.

38. (D)

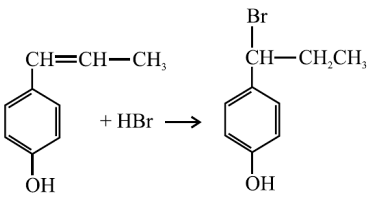
Sol.

$$CH_3-C\equiv CH \xrightarrow{Na} CH_3-C\equiv C^- Na^+$$

$$\xrightarrow[SN_2]{CH_3-CH_2-I} CH_3-C\equiv C-CH_2-CH_3$$

39. (A)

Sol.



40. (C)

Sol. Alkene on reaction with $SOCl_2$ doesn't give alkyl halide.

41. (B)

Sol. $CH_3CH_2CH_2OH \xrightarrow{PCl_5} CH_3CH_2CH_2Cl$

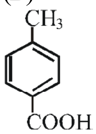
42. (A)

Sol. More positive charge on carbon then more feasible the attack of Nu–.

$H_2C=O > RCHO > ArCHO > R_2C=O > Ar_2C=O$

43. (B)

Sol.



(Due to presence of $-CH_3$)

44. (C)

Sol. 4-bromophenol as major product and 2- bromophenol is formed as minor product.

45. (C)

Sol. C – O – C angle in ethers is about 110°.

46. (C)

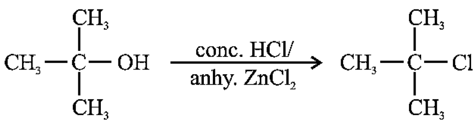
Sol. If $-NO_2$ is present on the benzene ring of phenols acidic strength increases particularly when that group is at ortho (or) para positions.

47. (A)

Sol. Alkyl halides on hydrolysis gives alcohols.

48. (A)

Sol.



49. (C)

Sol. $R-Br+Mg\xrightarrow{\text{dry ether}}R-MgBr$

Grignard reagent

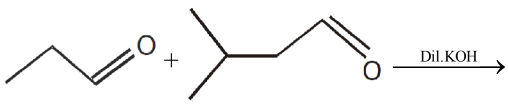
$\therefore$ It is a single bond organometallic compound.

50. (B)

Sol. Sodium hydrogen carbonate ($NaHCO_3$) being an alkali solution will dissolve an acid not a base so o-nitrophenol being a very weak acid than $NaHCO_3$ will not get dissolve.

51. 4

Sol.



Both aldehyde having α– H so 4 product will be formed in which two self and two cross products will be obtained.

52. 4

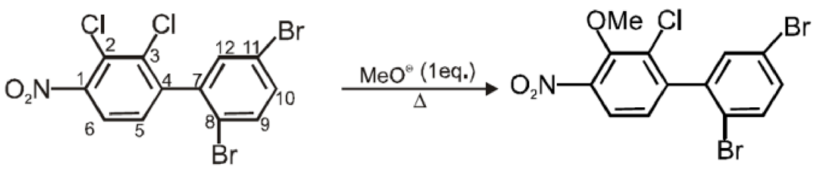
Sol. The electron pair at position (4) is not delocalized other are delocalized, hence position 4 is strongest nucleophile

53. 2

54. 3

55. 2

Sol.



56. 5

Sol.

$$CH_3-CH_2-\underset{CH_3}{\overset{Br}{C}}-CH_2-CH_2-CH_3 \xrightarrow[(-HBr)]{\text{alc. KOH}}$$

$$CH_3CH=\underset{CH_3}{C}-CH_2-CH_2-CH_3$$

cis + trans = 2 product

$$CH_3-CH_2-\underset{CH_3}{C}=CH-CH_2-CH_3$$

cis + trans = 2products

$$CH_3-CH_2-\underset{\|\atop CH_2}{C}-CH-CH_2-CH_3$$

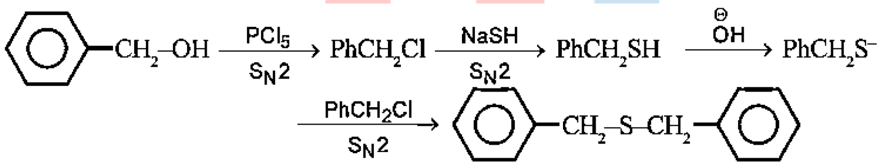
1 product

57. 4

Sol. 4(ii), (iv), (v), (vi) are only enols

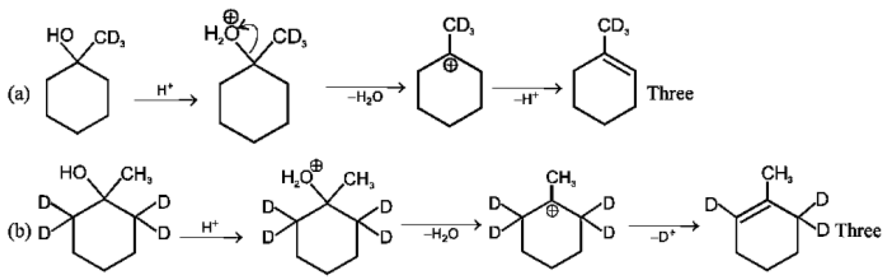
58. 3

Sol.



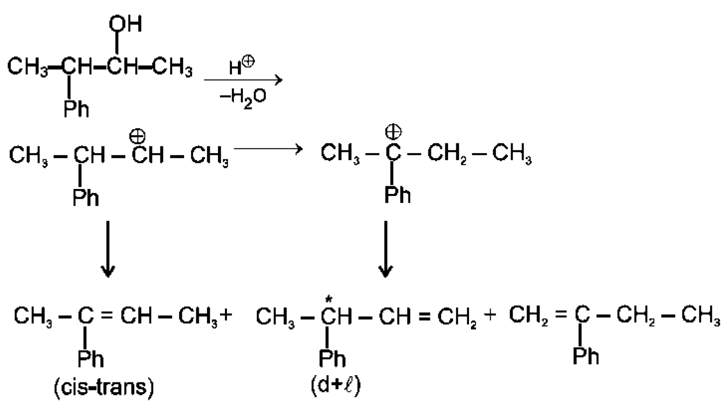
59. 6

Sol.



60. 5

Sol.



61. (D)

Sol. Solving the homogeneous equation, by using $y = vx$, we find the solution $x + y = c(x^2 + y^2)$

$y(-1) = 1 \Rightarrow x + y = 0$ which is a straight line.

62. (A)

Sol. $2x\cos y\,dx + y^2\cos x\,dx + 2y\sin x\,dy - x^2\sin y\,dy = 0$

$\Rightarrow (2x\cos y\,dx - x^2\sin y\,dy) +$

$(y^2\cos x\,dx + 2y\sin x\,dy) = 0$

$\Rightarrow d(x^2\cos y + y^2\sin x) = 0$

$\Rightarrow \quad x^2\cos y + y^2\sin x = C$

63. (A)

Sol. Equation of tangent at the point

R (x, f(x)) is $\qquad Y - f(x) = f'(x)(X - x)$

Coordinate of point P is $(0, f(x) - x f'(x))$

The slope of the perpendicular line through 'P' is

$$\frac{f(x) - xf'(x)}{-1} = -\frac{1}{f'(x)}$$

$$y\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^2 = 1$$ is differential equation.

64. (A)

Sol. Rearrange the diff. equation

$$xdx + \frac{ydx - xdy}{y^4} = 0$$

$$x^3dx + \frac{x^2}{y^2}\cdot\frac{ydx - xdy}{y^2} = 0$$

$$x^3dx + \left(\frac{x}{y}\right)^2 \frac{d}{dx}\left(\frac{x}{y}\right) = 0$$

integratings $\frac{x^4}{4} + \frac{1}{3}\left(\frac{x}{y}\right)^3 + c$

65. (C)

Sol. $y = u^m$

$dy/dx = mu^{m-1}\frac{du}{dx}$

The given differential equation becomes

$2x^4.u^m.\ mu^{m-1}\frac{du}{dx} + u^{4m} = 4x^6$

$$\Rightarrow \frac{du}{dx} = \frac{4x^6 - u^{4m}}{2mx^4x^{2m-1}}$$

For homogeneous equation degree should be same in numerator & denominator so,

$6 = 4m = 4 + 2m - 1 \Rightarrow m = 3/2$

66. (B)

Sol. Solving the equations of the asymptotes the centre is $x = 1$ and $y = 0$, since $e = \sqrt{2}$ the equation of the family of the hyperbola is

$$\frac{(x-1)^2}{a^2} - \frac{(y)^2}{a^2} = 1$$

$$\Rightarrow 2(x-1) - 2y\frac{dy}{dx} = 0$$

$\Rightarrow (x - 1) = yy'$ is differential equation.

67. (B)

Sol. Let the curve be $y = f(x)$. The equation of tangent at any point (x, y) is given by

$Y - y = f'(x)(X - x)$. So the portion of the axis of x which is cut off between the origin and the tangent at any point is obtained by putting $Y = 0$. Therefore,

$$x - \frac{y}{f'(x)} = Ky \Rightarrow x - y\frac{dx}{dy} = Ky \Rightarrow \frac{dx}{dy} - \frac{x}{y} = -K$$

which is a linear equation in x, so its integrating factor is $e^{-\int (1/y)dy} = y^{-1}$. Therefore, multiplying by y^{-1}, we have

$$\frac{d}{dy}(xy^{-1}) = -Ky^{-1} \Rightarrow xy^{-1} = -K\log y + C$$

$\Rightarrow x = y(C - K\log y)$

where C is arbitrary constant.

68. (C)

Sol. Since f(x) and g (x) are solution of given differential equation

$a f''(x) + x^2 f'(x) + f(x) = e^x$ ....(i)

& $ag''(x) + x^2 g'(x) + g(x) = e^x$ ....(ii)

(i) – (ii) given

$a[f''(x) - g''(x)] + x^2[f'(x) - g'(x)]$

$+ [f'(x) - g'(x)] = 0$

69. (B)

Sol. $$\int \frac{d\left(\frac{dy}{dx}\right)}{\frac{dy}{dx}} = \int \frac{dy}{y}$$

$$\therefore \ell n\left(\frac{dy}{dx}\right) = \ell ny + C$$

$$\therefore \frac{1}{y}\frac{dy}{dx} = C_1 \text{ i.e. } \frac{dy}{y} = C_1\,dx$$

$\ell ny = C_1x + C_2$

$\therefore y = e^{C_1x + C_2} = ke^{C_1x}$

70. (B)

Sol. $$\frac{x^2(x\,dx + y\,dy)}{\sqrt{x^2 + y^2}} = ydx - x\,dy$$

$$\text{or } \int \frac{1}{2}\frac{d(x^2 + y^2)}{\sqrt{x^2 + y^2}} = \int \frac{ydx - xdy}{x^2}$$

$$\Rightarrow \frac{1}{2}2.\sqrt{x^2 + y^2} = -\frac{y}{x} + C$$

71. (B)

Sol. 7^{α} always ends with 7, 9, 3, 1.

$\therefore 7^{\alpha} + 7^{\beta}$ is divisible by 5

$\Rightarrow$ Favourable events $= (25 \times 25) \times 2$

and sample space $= 100 \times 100$

$$\therefore \text{Required probability} = \frac{2(25 \times 25)}{100 \times 100} = \frac{1}{8}$$

72. (A)

Sol. Let $A = \{2, 3, 5\}$, $S = \{1, 2, 3, 4, 5, 6\}$

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

73. (B)

Sol. A : Event that first man speaks truth
B : Event that second man speaks truth
R : Day is rainy

$$P(R) = \frac{P(A \cap B).P(R)}{P(A \cap B).P(R) + P(A' \cap B').P(R')}$$

$$= \frac{\frac{4}{5}.\frac{2}{3}.\frac{3}{4}}{\frac{4}{5}.\frac{2}{3}.\frac{3}{4} + \frac{1}{5}.\frac{1}{3}.\frac{1}{4}} = \frac{24}{25}$$

74. (C)

Sol. Given set of numbers is {1, 2, ..., 9} in which 4 are even and 5 are odd, so in the given product it is not possible to arrange to substract only even number from odd number, there must be atleast one factor involving substraction of an odd number from another odd number. So at least one of the factor is even. Hence product is always even.
$\therefore$ Required probability = 1

75. (B)

Sol. Here, $n(S) = 4 \times 4 \times 4 = 64$
and n(A) = 211 or 312 or 413 or 431 or 422

$$\therefore \quad P(E) = \frac{6}{64} = \frac{3}{32} = \frac{3}{3+29}$$

Odds infavour3 : 29.

76. (D)

Sol. Let X denote the largest number on the n tickets drawn. We have

$$P(X \le k) = \left(\frac{k}{N}\right)^n \text{ and } P(X \le k-1) = \left(\frac{k-1}{N}\right)^n$$

$$\text{Thus } P(X = k) = \left(\frac{k}{N}\right)^n - \left(\frac{k-1}{N}\right)^n$$

77. (A)

Sol. Let us first count the number of elements in F. Total number of functions from A to B is $3^4 = 81$.
The number of functions which do not contain x(y) [z] in its range is 2^4.
$\therefore$ the number of functions which contain exactly two elements in the range is $3 \cdot 2^4 = 48$.
The number of functions which contain exactly one element in its range is 3.
Thus, the number of onto functions from A to B is 81 – 48 + 3 = 36
[using principle of inclusion exclusion]
n (F) = 36.
Let $f \in F$. We now count the number of ways in which $f^{-1}(x)$ consists of single element.
We can choose preimage of x in 4 ways. The remaining 3 elements can be mapped onto {y, z} is $2^3 - 2 = 6$ ways.
$\therefore$ f^{-1} (x) will consists of exactly one element in $4 \times 6 = 24$ ways.
Thus, the probability of the required event is
24/36 = 2/3

78. (A)

Sol. Let E_1 denote the event that the letter came from TATANAGAR and E_2 the event that the letter came from CALCUTTA. Let A denote the event that the two consecutive alphabets visible on the envelope are TA. We have $P(E_1) = 1/2$, $P(E_2) = 1/2$, $P(A / E_1) = 2/8$, $P(A / E_2) = 1/7$. Therefore, by Bayes' theorem we have

$$P(E_2 / A) = \frac{P(E_2)P(A/E_2)}{P(E_1)\ P(A/E_1) + P(E_2)\ P(A/E_2)}$$

$$= \frac{4}{11}$$

79. (B)

Sol. $\frac{4}{13} = \frac{|a-3|}{13} \Rightarrow = 7 \text{ or } -1$

80. (B)

Sol. n (S) = total number of ways in which 10 persons can be made to stand = 10!
n (E) = Number of ways in which exactly one person stands between A and B
= (10 – 2) . 2. (10 – 2)! = 16 × 8!

$\therefore P(E) = \frac{16 \times 8!}{10!} = \frac{16}{10 \times 9} = \frac{8}{45}$

81. 82

Sol. no. of numbers = 5 ! = 120
E = Event that the number is divisible by 4.
So last two digits to see **12**, 13, 14, 15
21, 23, **24**, 25
31, **32**, 34, 35
41, 42, 43, 45
51, **52**, 53, 54
of the above 4 nos are divisible by 4
Son(E) = no. of numbers divisible by 4
= 3!.4 = 24

$\text{Probability} = \frac{n(E)}{n(S)} = \frac{24}{120} = \frac{1}{5} = 0.2$

$\Rightarrow$ integral part of $(\sqrt{2} + 1)^5$ will be 82.

82. 375

Sol. Let
E = the event that A reports it is six
E_1 = six turns up when a dice is thrown
E_2 = six does not turns up when a dice is thrown
Probability that six actually showed up if A reported that of was a six = $P(E_1/E)$

$P(E_1/E) = \frac{P(E_1 \cap E)}{P(E)}$

$= \frac{P(E_1).P(E_1/E)}{P(E_1).P(E_1/E) + P(E_2).P(E_2/E)}$

$P(E/E_1)$ = Probability that the person reports six when it actually showed
= Probability that he speak thruth
= 3/4

$P(E/E_2)$ = Probability that the person reports six when it did not show six

= Probability that he lies = $1 - \frac{3}{4}$

$= \frac{1}{4}$

also $P(E) = \frac{1}{6}$ and $P(E_2) = \frac{5}{6}$

$P(E_1/E) = \frac{\frac{1}{6}.\frac{3}{4}}{\frac{1}{6}.\frac{3}{4} + \frac{5}{6}.\frac{1}{4}} = \frac{\frac{1}{8}}{\frac{1}{8} + \frac{5}{24}} = \frac{3}{8}$

$\Rightarrow \quad 1000P = 1000.\frac{3}{8} = 375$

83. 2

Sol. Total number of arrangements = 15!

Extreme chairs are occupied by girls, thus there are four gaps among 5 girls where boys can be seated the number of boys in these four gaps be 2x + 1, 2y + 1, 2z + 1 and 2t + 1, then

$2x + 1 + 2y + 1 + 2z + 1 + 2t + 1 = 10$

$\Rightarrow \quad x + y + z + t = 3$

Where x, y, z, t are integers and $0 \le x \le 3$, $0 \le y \le 3$, $0 \le z \le 3$, $0 \le t \le 3$

$\therefore$ The number of ways of selecting positions for boys = coefficient of x3 in (1 + x + x2 + x3)4 = coefficient of x3 in

$$\left(\frac{1-x^4}{1-x}\right)^4$$

= coefficient of x3 in (1– x4)4 (1 – x)–4 = 6C3 = 20

$\therefore$ Number of arrangements of boys and girls with given condition = 20 × 20! × 5!

$$\therefore \quad \text{Required probability} = \frac{20\times 10!\times 5!}{15!} = \frac{20}{3003}.$$

84. 2

Sol. P(A1) = P(ω) + P(BBω) + P(BBBBω)

$$= \frac{3}{10} + \frac{5}{10}.\frac{4}{9}.\frac{3}{8} + \frac{5}{10}.\frac{4}{9}.\frac{3}{8}.\frac{2}{7}.\frac{3}{6}$$

$$= \frac{3}{10} + \frac{1}{12} + \frac{1}{12\times 7} = \frac{332}{840} = \frac{83}{210}$$

$\Rightarrow \quad$ P(A2) = P(Bω) + P(BBBω) + P(BBBBBω)

$$= \frac{5}{10}.\frac{3}{9} + \frac{5}{10}.\frac{4}{9}.\frac{3}{8}.\frac{3}{7} + \frac{5}{10}.\frac{4}{9}.\frac{3}{8}.\frac{2}{7}.\frac{1}{6}.\frac{3}{5}$$

$$= \frac{1}{6} + \frac{1}{28} + \frac{1}{420} = \frac{86}{420} = \frac{43}{210}$$

P(B) = P(R) + P(BR) + P(BBR) + P(BBBR) + P(BBBBR) + P(B5R)

$$\frac{2}{10} + \frac{5}{10}.\frac{2}{9} + \frac{5}{10}.\frac{4}{9}.\frac{2}{8} + \frac{5}{10}.\frac{4}{9}.\frac{3}{8} + \frac{2}{7} +$$

$$\frac{5}{10}.\frac{4}{9}.\frac{3}{8}.\frac{2}{7}.\frac{2}{6} + \frac{5}{10}.\frac{4}{9}.\frac{3}{8}.\frac{2}{7}.\frac{1}{6}.\frac{2}{5}$$

$$= \frac{1}{5} + \frac{1}{9} + \frac{1}{18} + \frac{1}{42} + \frac{1}{26} + \frac{1}{630}$$

$$= \frac{126+70+35+15+5+1}{630} = \frac{2\times 126}{630} = \frac{2}{5}$$

85. 2

Sol. Prob. of selection of way box is $= \frac{1}{N+1}$

Let E be the event that the wall clock selected is effective then

P(E) = P(B1) P(E/B1) + P(B2) P(E/B2) + ... + P(BN + 1) P(E/BN + 1)

$$= \frac{1}{N+1}\left[1 + \frac{N-1}{N} + \frac{N-2}{N} + ...\frac{1}{N} + 0\right]$$

$$= \frac{1+2+.....+N}{N(N+1)} = \frac{1}{2}$$

$$P(B1/E) = \frac{P(B_k).P(E/B_k)}{RE} = \frac{\frac{1}{N+1}\frac{(N-K+1)}{N}}{1/2}$$

$$= \frac{2N-2K+2}{N^2+N}$$

86. 48

Sol. Let Population = x, time = t (in years)

Given $\frac{dx}{dt} \propto x \Rightarrow \frac{dx}{dt} = kx$

Where k is a constant of proportionality

or $\frac{dx}{x} = k\,dt$

Integrating, we get

$ln\, x = kt + ln\, c$

$\Rightarrow ln\left(\frac{x}{c}\right) = kt \Rightarrow \frac{x}{c} = e^{kt}$

or $x = ce^{kt}$

If initially i.e., when time $t = 0$, $x = x_0$

then $x_0 = ce^0 = c$

$\therefore\ x = x_0 e^{kt}$

Given $x = 2x_0$, $t = 30$

then $2x_0 = x_0 e^{30k} \quad \Rightarrow 2 = e^{30k}$

$\therefore ln\, 2 = 30\, k \quad ...(1)$

To find t, when it triples, $x = 3x_0$

$\therefore\ 3x_0 = x_0 e^{kt} \quad \Rightarrow 3 = e^{kt}\ ...(2)$

Dividing (2) by (1) then $\frac{t}{30} = \frac{\ln 3}{\ln 2}$

or $t = 30 \times \frac{\ln 3}{\ln 2} = 30 \times 1.5849 = 48$ years.

87. 40

Sol. Let T be the temperature of the substance at a time t then $-\frac{dT}{dt} \propto (T - 290)$

$\Rightarrow \frac{dT}{dt} = -k\,(T - 290)$

Where k is constant of proportionality and negative sign denote rate of cooling.

or $\frac{dT}{(T - 290)} = -k\,dt$

integrating, we get

$\int \frac{dT}{(T - 290)} = -k \int dt$

$\Rightarrow ln\,(T - 290) = -kt + ln\, c$

$\Rightarrow \left(\frac{T - 290}{c}\right) = e^{-kt}$

or $(T - 290) = ce^{-kt}$

If initially i.e., when $t = 0$ & $T = 370$

Then $(370 - 290) = ce$

$\therefore\ c = 80$

$\therefore\ T - 290 = 80e^{kt}\ ...(1)$

and for $t = 10$, $T = 330$

$\therefore\ 330 - 290 = 80e^{-10k} \quad \Rightarrow\ (40) = 80e^{-10k}$

$\Rightarrow 2 = e^{10k}$

$ln 2 = 10\, k \quad ...(2)$

To find t, when T = 295

from (1), $295 - 290 = 80e^{-kt}$

$\Rightarrow \dfrac{5}{80} = e^{-kt}$ $\qquad \Rightarrow ln\, 16 = kt$

or $\quad 4\, ln\, 2 = kt$ $\qquad$ ... (3)

Dividing (3) by (2) then $4 = \dfrac{t}{10}$

$\therefore$ t = 40 minutes.

88. 79

Sol. Let x denote the population at a time t in years.

then $\dfrac{dx}{dt} \propto x \Rightarrow \dfrac{dx}{dt} = kx$

when k is a constant of proportionality.

Solving $\dfrac{dx}{dt} = kx$, we get

$\int \dfrac{dx}{x} = \int k\, dt \Rightarrow \log x = kt + c$

$\Rightarrow x = e^{kt + c}$

$\Rightarrow x = x_0 e^{kt}$

Where x_0 is the population at time t = 0.

Since it doubles in 50 years, at t = 50, we must have $x = 2x_0$.

Hence $2x_0 = x_0 e^{50k} \Rightarrow 50k = \log 2$

$\Rightarrow k = \dfrac{\log 2}{50}$ so that $x = x_0 e^{\frac{\log 2}{50} t}$

To find t, when it triples, $x = 3x_0$

$\Rightarrow 3x_0 = x_0 e^{\frac{\log 2}{50} t} \Rightarrow \log 3 = \dfrac{\log 2}{50} t$

$\Rightarrow t = \dfrac{50 \log 3}{\log 2} = 79$ years.

89. 3

Sol. $y = Ax + A^3$

$\dfrac{dy}{dx} = A$

Equation $y = \dfrac{xdy}{dx} + \left(\dfrac{dy}{dx}\right)^3$ Degree = 3

90. 3

Sol. $\dfrac{dy}{dx} = y + 1$

variable sep.

$\int \dfrac{dy}{y+1} = \int dx \Rightarrow \log(y+1) = x + c,\ y(0) = 1$

$\log 2 = c$

So $\log(y + 1) = x + \log 2$

$\Rightarrow y\ (\ell n\ 2)$

$\log(y + 1) = 2 \log 2 = \log 4$

$y + 1 = 4$

$\Rightarrow y = 3$