JEE MAIN ANSWER KEY & SOLUTION

PAPER CODE :- PART TEST-5
CLASS-XI

ANSWER KEY

PHYSICS

1.	(C)	**2.**	(A)	**3.**	(C)	**4.**	(D)	**5.**	(A)	**6.**	(B)	**7.**	(C)
8.	(C)	**9.**	(B)	**10.**	(C)	**11.**	(D)	**12.**	(B)	**13.**	(C)	**14.**	(D)
15.	(D)	**16.**	(D)	**17.**	(D)	**18.**	(C)	**19.**	(A)	**20.**	(B)	**21.**	16
22.	17	**23.**	10	**24.**	50	**25.**	6	**26.**	7	**27.**	55	**28.**	2
29.	4	**30.**	8										

CHEMISTRY

31.	(B)	**32.**	(D)	**33.**	(C)	**34.**	(B)	**35.**	(D)	**36.**	(D)	**37.**	(D)
38.	(D)	**39.**	(A)	**40.**	(D)	**41.**	(A)	**42.**	(D)	**43.**	(D)	**44.**	(B)
45.	(C)	**46.**	(C)	**47.**	(B)	**48.**	(C)	**49.**	(C)	**50.**	(B)	**51.**	4
52.	4	**53.**	21	**54.**	6	**55.**	8	**56.**	6	**57.**	12	**58.**	13
59.	4	**60.**	6										

MATHEMATICS

61.	(D)	**62.**	(A)	**63.**	(A)	**64.**	(B)	**65.**	(C)	**66.**	(B)	**67.**	(C)
68.	(B)	**69.**	(D)	**70.**	(B)	**71.**	(C)	**72.**	(A)	**73.**	(C)	**74.**	(C)
75.	(D)	**76.**	(A)	**77.**	(B)	**78.**	(B)	**79.**	(C)	**80.**	(C)	**81.**	136
82.	100	**83.**	937	**84.**	8	**85.**	10	**86.**	0	**87.**	3	**88.**	14
89.	63	**90.**	6										

PHYSICS

1. (C)

Sol. Let both P_1 and P_2 meet at distance x_0 from the bottom at time 't'.

For P_2: $v = \sqrt{\frac{T}{\mu}} \Rightarrow \frac{dx}{dt} = \sqrt{gx} \Rightarrow \int_0^t dt = \int_0^{x_0} \frac{dx}{\sqrt{gx}}$

$\Rightarrow t_1 = \sqrt{\frac{4x_0}{g}}$

For P_1: $t_2 = \sqrt{\frac{4\ell}{g}} - \sqrt{\frac{4x_0}{g}}$

$\because t_1 = t_2 \Rightarrow x_0 = \frac{\ell}{4}$

2. (A)

Sol. $T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{\ell}+\frac{1}{R}\right)}} = 2\pi\sqrt{\frac{R}{2g}}$

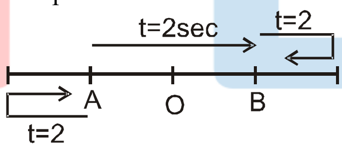
$T = \pi\sqrt{\frac{2R}{g}}$

3. (C)

Sol. Since slope is constant, therefore velocity of particle is also constant. Hence there will be no acceleration.

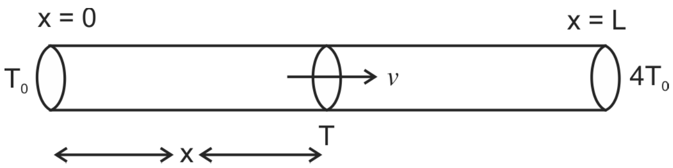
4. (D)

Sol. From the given information it can be inferred that points A and B are equidistant from mean position
Hence from diagram it is clear that time period of oscillation is $= 2 + 2 \times 2 + 2 = 8$ second.



5. (A)

Sol.



$T = \frac{3T_0}{L}X + T_0$

Wave velocity

$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\gamma R}{M}} \cdot \sqrt{\frac{3T_0 X}{L} + T_0} = \frac{dx}{dt}$

$\int_0^t dt = \sqrt{\frac{M}{\gamma R}} \int_0^L \frac{dx}{\sqrt{\frac{3T_0 X}{L} + T_0}}$

$\Rightarrow t = \sqrt{\frac{4ML^2}{15RT_0}}$

6. (B)

Sol. Equation of the component waves are :

$y = A\sin(\omega t - kx)$ and $y = A\sin(\omega t + kx)$

where; $\omega t - kx$ = constant or $\omega t + kx$ = cosntant

Differentiating w.r.t. 't' ;

$$\omega - k\frac{dx}{dt} = 0 \quad \text{and} \quad \omega + k\frac{dx}{dt} = 0$$

$$\Rightarrow v = \frac{dx}{dt} = \frac{\omega}{k} \quad \text{and} \quad v = -\frac{\omega}{k}$$

i.e.; the speed of component waves is $\left(\frac{\omega}{k}\right)$. Hence (B)

7. (C)

Sol. Conceptual

8. (C)

Sol. $f = \frac{5V}{2\ell} = 5\text{ Hz}$

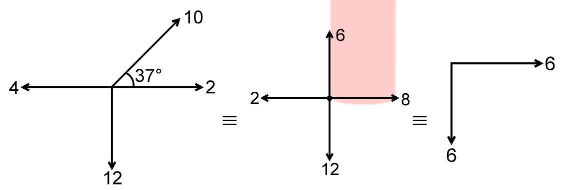
9. (B)

Sol. (A) particles between two consecutive nodes are only in same phase.

(C) Amplitude is maximum at antinodes and minimum at nodes.

10. (C)

Sol. Amplitude phasor diagram :



$\therefore$ resultant amplitude = $6\sqrt{2}$.

11. (D)

Sol. $E = \frac{1}{2}mA^2\omega^2$ i.e., $E \propto (A\omega)^2$

or $(A_1\omega_1)^2 = (A_2\omega_2)^2$

or $A_1\omega_1 = A_2\omega_2$

or $4 \times 10 = 5 \times \omega$

or $\omega = 8$ unit

12. (B)

Sol. $A\omega = v_{max}$

$$T = \frac{2\pi}{\omega} = \frac{2\pi A}{v_{max}} = 0.01 \text{ sec.}$$

13. (C)

Sol. even as well as odd harmonics are produced.

14. (D)

Sol. $x = x_0 \cos\left(\omega t - \frac{\pi}{4}\right)$, $\quad v = -x_0 \omega \sin\left(\omega t - \frac{\pi}{4}\right)$

$a = -x_0 \omega^2 \cos\left(\omega t - \frac{\pi}{4}\right)$, $\quad a = x_0 \omega^2 \cos\left(\omega t - \frac{\pi}{4} + \pi\right)$

$a = x_0 \omega^2 . \cos\left(\omega t + \frac{3\pi}{4}\right)$

15. (D)

Sol. $f = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$, $f_{new} = \frac{1}{2\pi}\sqrt{\frac{4k_1 + 4k_2}{m}} = 2f.$

16. (D)

Sol. By definition

17. (D)

Sol. C.O.L.M.

$MV_{max} = (m + M)V_{new}$, $V_{max} = A_1 \omega_1$

$V_{new} = \frac{MV_{max}}{(m + M)}$

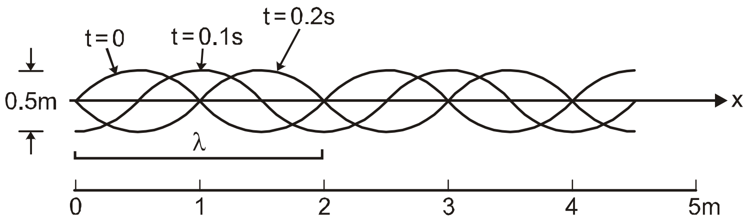
Now , $V_{new} = A_2 . \omega_2$

$\frac{M.A_1}{(m+M)}\sqrt{\frac{K}{M}} = A_2\sqrt{\frac{K}{(m+M)}}$

$A_2 = A_1\sqrt{\frac{M}{(m+M)}}$ $\qquad \frac{A_1}{A_2} = \left(\frac{m+M}{M}\right)^{1/2}$ **Ans.**

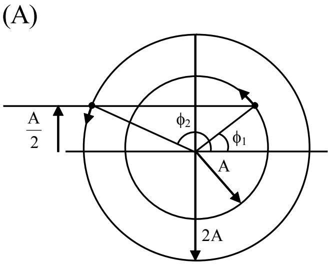
18. (C)

Sol. Clear from the figure



19. (A)

Sol.



$\sin \phi_1 = \frac{A/2}{A} = \frac{1}{2}$

$\phi_1 = \frac{\pi}{6}$

$\sin(\pi - \phi_2) = \frac{A/2}{2A} = \frac{1}{4}$

$$\phi_2 = \pi - \sin^{-1}\left(\frac{1}{4}\right)$$

Phase difference

$$\phi_2 - \phi_1 = \frac{5\pi}{6} - \sin^{-1}\left(\frac{1}{4}\right)$$

20. (B)

Sol. $R_A = \frac{V}{V_A}$, $R_B = \frac{V}{V_B}$

as $V_A > V_B$, $R_A < R_B$

21. 16

Sol. $\beta_1 = 10\log\frac{I}{I_0}$ $\beta_2 = 10\log\frac{4I}{I_0}$

$\beta_2 - \beta_1 = 10\log 4$

$\beta_2 = 10 + 10\log 4 = 10 + 6 = 16$

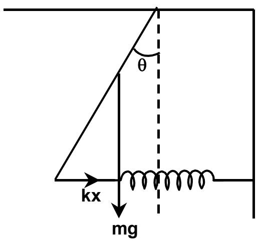
22. 17

Sol. Let the rod be displaced by a small angle θ from the equilibrium position. The torque of the restoring spring force

$$\tau_1 = K(L\theta)L\cos\theta = KL^2\theta \qquad [\cos\theta \cong 1]$$

The torque of the weight $\tau_2 = mg\frac{L}{2}\sin\theta = mg\frac{L}{2}\theta$ $[\sin\theta \approx \theta]$

$\therefore$ Total torque $\tau = \tau_1 + \tau_2 = \left(KL^2 + \frac{mgL}{2}\right)\theta$



Since torque is against angular displacement

$$I\alpha = -\left(KL^2 + \frac{mgL}{2}\right)\theta$$

$$\therefore \frac{d^2\theta}{dt^2} = -\left(\frac{KL^2}{I} + \frac{mgL}{2I}\right)\theta$$

$$\therefore \omega = \left(\frac{3K}{m} + \frac{3g}{2L}\right)^{1/2} = \left[3\left(\frac{K}{m} + \frac{g}{2L}\right)\right]^{1/2}$$

$$\therefore \text{Frequency} = \frac{1}{2\pi}\sqrt{3\left(\frac{K}{m} + \frac{g}{2L}\right)}$$

23. 10

Sol. Total mechanical energy of vibration = $\int \frac{1}{2} dm\, v^2$

$dm = \mu\, dx$, $\mu = 10^3\ kg/m^3\ (0.04)\ m^2$

$\mu = 40\ kg/m$

$v(x) = \frac{ds}{dt} = 5\ mm \sin \pi x\ (-\sin(200\ t))$

Total mechanical energy $= \frac{1}{2}\mu \int v^2(x)dx = 10$ joules.

24. 50

Sol. $F_{max} = m\omega^2 A = 50\sqrt{2}$

KE = PE at $X = \frac{A}{\sqrt{2}}$

$F = \frac{m\omega^2 A}{\sqrt{2}} = 50$

25. 6

Sol. Equation of stationary wave from closed end = a sink x cos ωt

Now at $x = \frac{\ell}{7}$

3rd overtone

$\frac{7\lambda}{4} = \ell$

Amplitude $= a \sin kx = a \sin \frac{2\pi}{\lambda}\frac{\ell}{7}$

$= a \sin \frac{2\pi}{\lambda}\frac{7\lambda}{4\times 7} = a \sin \frac{\pi}{2} = a$

26. 7

Sol. $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow k = 4\pi^2 m \times f^2$

$k = 4\pi^2 \times \frac{108\times 10^{-3}}{6.02\times 10^{23}} \times f^2$

$= \frac{4\times \pi^2 \times 108}{6.02} \times \frac{10^{24}\times 10^{-3}}{10^{23}}$

$= 7.1\ N/m$

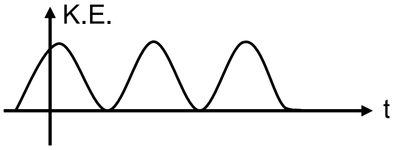
27. 55 cm

Sol. $\Delta n = \frac{V}{2\ell} = (2100 - 1800)$

$\ell = \frac{330}{2(300)}\ m = \frac{330\times 100}{2\times 300}\ cm = 55\ cm$

28. 2

Sol. $K.E. = \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t + \varphi)$

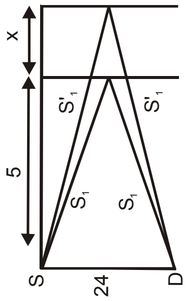


29. 4 cm

Sol. Initially path difference $2S_1 - 24 = \lambda/3$ ...(i)

$\Rightarrow$ where $\lambda = 6$

If shifting is x for bring waves in phase $2S_1' - 24 = \lambda$ ...(ii)



subtract (2) and (1)

$$2S_1' - 2S_1 = \frac{2\lambda}{3}$$

$$S_1' - S_1 = \frac{\lambda}{3}$$

$$S_1' - S_1 = \frac{6}{3}$$

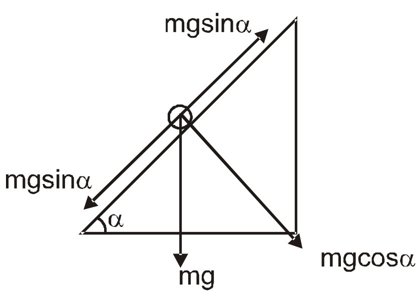
$$\sqrt{(5+x)^2 + 12^2} - \sqrt{5^2 + 12^2} = 2$$

on solving x = 4 cm

30. 8

Sol. Free body diagram of bob of the pendulum with respect to the accelerating frame of reference is as follows:

$\therefore$ Net tension in the string is $T = mg\cos\alpha$



So, $g_{eff} = \frac{T}{m} = \frac{mg\cos\alpha}{m} = g\cos\alpha$

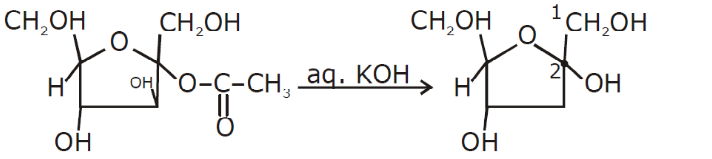
$$T = 2\pi\sqrt{\frac{L}{g_{eff}}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$$

31. (B)

Sol. Between double bond & triple bond double bond is preferred.

32. (D)

Sol.



2nd carbon → Anomeric carbon

33. (C)

Sol. The N of nitro group will be attached from carbon chain so alternate 2nd & 4th are wrong in this way & the alternate 1st is 3-nitro-1- propane amine.

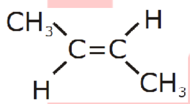
34. (B)

Sol. IUPAC name of carbonyl chloride is chloromethanoyl chloride.

(Chloromethanoyl chloride) Principal functional group is $\overset{\overset{\displaystyle O}{\|}}{-C-Cl}$ (-oyl-chloride)

Its common name is phosgene and it is poisonous gas.

35. (D)

Sol. $CH_3{-}C{\equiv}CH_3 \xrightarrow{Na/Liq.NH_3}$ 

Birch reduction

36. (D)

Sol. $C_4H_6O_2$ represents

$$H_3C-\underset{\underset{\displaystyle O}{\|}}{C}-\underset{\underset{\displaystyle O}{\|}}{C}-CH_3$$

A diketone

$$H-\underset{\underset{\displaystyle O}{\|}}{C}-CH_2-CH_2-\underset{\underset{\displaystyle O}{\|}}{C}-H$$

Compound having two aldehydes

$$H_3C-HC=CH-\underset{\underset{\displaystyle O}{\|}}{C}-OH$$

Alkenoic acid

37. (D)

Sol. Ring chain isomers have same molecular formula but isomers have a cyclic structure and open-chain structure. Cyclopropane is a ring chain isomer of propene as both have same molecular formula C_3H_6.
Cyclo alkanes are the isomers of alkene. Hence, option B is correct.
C6can form cyclohexane as well as hexene.Hence both are ring chian isomer of each other.

38. (D)

39. (A)

40. (D)

Sol. $-CH_3$ is electron donating group.

41. (A)

Sol. In – I group, the priority of $-{}^{+}N(CH_3)_3$ IS greater than other group.

42. (D)

43. (D)

Sol. $CH_3OH > HC \equiv CH > C_6H_6 > C_2H_6$

Although all are neutral towards litmus paper, the most acidic among them is methanol. Because the conjugate base formed by the loss of proton from methanol is most stable.

44. (B)

Sol. Heterolytic fission results in the formation of two different chemical species in the sense that one is a cation and the other an anion. Heterolysis of propane:

$CH_3CH_2CH_3 \quad CH_3^- + CH_3CH_2^+$

45. (C)

46. (C)

Sol. In the compound electrophilic substitution occurs at ortho para position at ring II.

–OCOPh group is activating and ortho para directing.

–COOPh group is deactivating and meta directing.

47. (B)

Sol. A compound can be divided into two equal halves and contains even n asymmetric carbon atoms. The number of stereo isomers is $2^{n-1} + 2^{(n/2-1)}$. 2^{n-1} represents the number of d, 1 isomers and $2^{(n/2-1)}$ represents the number of meso forms.

48. (C)

Sol. The correct IUPAC name of the given compound is N-ethyl methanamide.

It is an amide of formic acid with ethyl amine. The ethyl group is present as a substituent on the nitrogen atom.

49. (C)

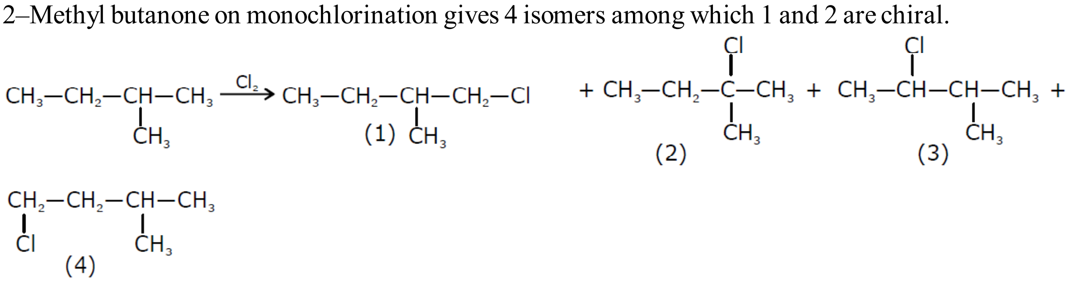
50. (B)

Sol. Since the rotation caused by chiral centre is cancelled by the other, it is a meso compound. Hence the optical rotation is 0°.

51. 4

52. 4

Sol. 2–Methyl butanone on monochlorination gives 4 isomers among which 1 and 2 are chiral.



53. 21

54. 6

55. 8

Sol. There are 8 structural isomers of $C_5H_{11}Br$

1-bromopentane
2-bromopentane
3-bromopentane
1-bromo-2-methylbutane
2-bromo-2-methylbutane
2-bromo-3-methylbutane
1-bromo-3-methylbutane
1-bromo-2,2-dimethylpropane.

56. 6

Sol. There is five isomers possible for $C_4H_6Br_2$ -

[1] 1,1di bromo cyclo butane
[2] 1.2 di bromo cyclo butane {Including optical isomer}
[3] 1.3 di bromo cyclo butane

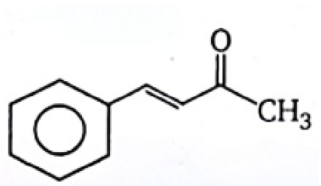
57. 12

Sol. Total no. of structural isomers of C_9H_{18} containing cyclohexane ring = 12.

58. 13

59. 4

Sol.



Parent carbon chain is the longest carbon chain present in the carbon as shown in the above image. Here, the longest carbon chain has 8 carbon atoms.

60. 6

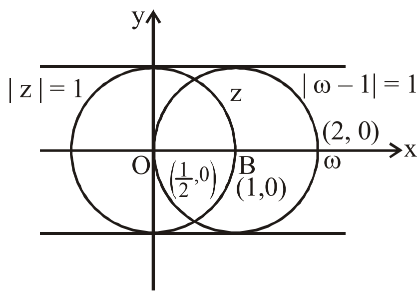
Sol. Chiral is a molecule or ion is superimposable on its mirror image and all the substituent attached to the carbon should be different. Here 6 carbons are chiral on which all the substituent attached are different.

61. (D)

Sol. $E = |2z-1|^2 + |2\omega - 1|^2$

$$E = 4\left[\left|z-\frac{1}{2}\right|^2 + \left|\omega - \frac{1}{2}\right|^2\right]$$

$\left|z-\frac{1}{2}\right|$ is the distance of z from $\left(\frac{1}{2}, 0\right)$ and $\left|\omega-\frac{1}{2}\right|$ in the distance of ω from $\left(\frac{1}{2}, 0\right)$.



$$E_{max} = 4\left[\left(-1-\frac{1}{2}\right)^2 + \left(2-\frac{1}{2}\right)^2\right] = 4\left[\frac{9}{4}+\frac{9}{4}\right] = 18$$

$$E_{min} = 4\left[\left(1-\frac{1}{2}\right)^2 + \left(0-\frac{1}{2}\right)^2\right] = 4\left[\frac{1}{4}+\frac{1}{4}\right] = 2$$

Hence, $E = [2, 18]$

62. (A)

Sol. Let α is a non real complex root of unity that is also a root of the equation $z^2 + az + b = 0$, then $\overline{\alpha}$ will also be its root. $(|\alpha| = 1)$

Hence $\alpha + \overline{\alpha} = -a$

$\therefore$ $|a| = |\alpha + \overline{\alpha}| \le |\alpha| + |\overline{\alpha}| = 2$

and $b = \alpha\overline{\alpha} = 1$

Hence we must check those equation for which $-2 \le a \le 2$ and $b = 1$

i.e. $z^2 + 2z + 1 = 0$; $z^2 + z + 1 = 0$; $z^2 + 1 = 0$

$z^2 - 2z + 1 = 0$; $z^2 - z + 1 = 0$

hence roots are $\pm 1, \pm i$; $\dfrac{-1 \pm \sqrt{-3}}{2}, \dfrac{1 \pm \sqrt{-3}}{2}$ i.e. 8

two roots are rejected becasues they are real. hence 6. **Ans.**]

63. (A)

Sol. l_1 is perpendicular to l_2

$\Rightarrow$ $\dfrac{z_1 - z_2}{z_3 - z_4}$ is purely imaginary

$$\frac{z_1 - z_2}{z_3 - z_4} + \frac{\overline{z}_1 - \overline{z}_2}{\overline{z}_3 - \overline{z}_4} = 0$$

$$\frac{z_1 - z_2}{\overline{z}_1 - \overline{z}_2} + \frac{z_3 - z_4}{\overline{z}_3 - \overline{z}_4} = 0 \Rightarrow \omega_1 + \omega_2 = 0$$

Note: If l_1 parallel to l_2 then

$$\frac{z_1 - z_2}{z_3 - z_4} = \frac{\overline{z}_1 - \overline{z}_2}{\overline{z}_3 - \overline{z}_4} \Rightarrow \omega_1 = \omega_2]$$

64. (B)

Sol. Let ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Latus rectum : $\frac{2b^2}{a}=\frac{5}{2}$

$\frac{2\left(a^2(1-e^2)\right)}{a}=\frac{5}{4} \quad \Rightarrow a\left(1-\frac{1}{4}\right)=\frac{5}{4} \Rightarrow a=\frac{5}{3}$

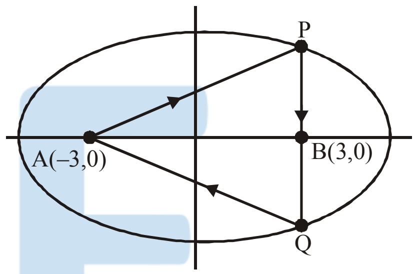
But $\frac{2b^2}{a}=\frac{5}{2} \quad \Rightarrow \frac{b^2}{a}=\frac{5}{4}$

$b^2=\frac{5}{4}\times\frac{5}{3}=\frac{25}{12}$

$\therefore \frac{x^2}{\frac{25}{9}}+\frac{y^2}{\frac{25}{2}}=1 \Rightarrow \frac{9x^2}{25}+\frac{12y^2}{25}=1.$]

65. (C)

Sol. Ellipse is $\frac{x^2}{25}+\frac{y^2}{16}=1.$



$\therefore a=5, b=4$

$\therefore e=\sqrt{1-\frac{b^2}{a^2}}=\frac{3}{5}$

$\therefore$ Foci $=(\pm 3, 0)$

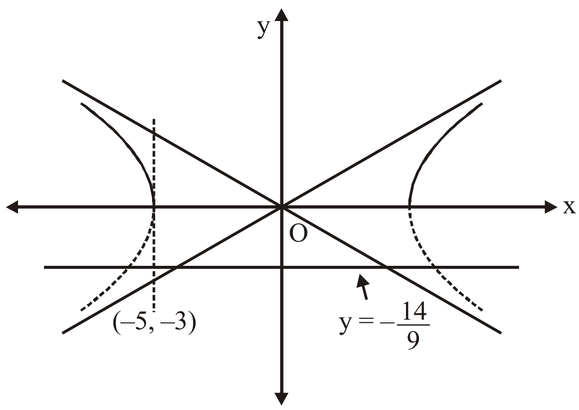
$\because$ Reflected ray is parallel to minor axis i.e. y-axis.

$\therefore P=\left(3,\frac{16}{5}\right)$ and $Q=\left(3,\frac{-16}{5}\right)$

$\therefore$ Area of $\Delta APQ=\frac{1}{2}\times 6\times\frac{32}{5}=\frac{96}{5}.$]

66. (B)

Sol. $y=\frac{3}{5}\sqrt{x^2-25}$



$25y^2=9(x^2-25)$

$9x^2-25y^2=9\times 25$

$\frac{x^2}{25}-\frac{y^2}{9}=1$

Equation of asymptotes are $y=\pm\frac{3}{5}x$

$y=\frac{-14}{9}=\pm\frac{3}{5}x \Rightarrow x=\frac{-70}{27}$

$\therefore$ For two distinct tangents to the curve $y=\frac{3}{5}\sqrt{x^2-25}$ from the point $\left(a,\frac{-14}{5}\right)$ can be drawn

if $a\in\left[\frac{-70}{27},\frac{70}{27}\right]$

Hence, number of integral values of 'a' are 5. **Ans.**]

67. (C)

Sol. Hyperbola be $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$

$B^2 = A^2(e^2 - 1) = a^2e'^2 - A^2 = (a^2 - b^2) - A^2$

$\therefore$ Hyperbola is $\frac{x^2}{A^2} + \frac{y^2}{A^2 - (a^2 - b^2)} = 1$

$\because$ $\left(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$ lies on it

$\therefore$ $\frac{a^2}{2A^2} + \frac{b^2}{2(A^2 - a^2 + b^2)} = 1$

$\Rightarrow$ $a^2 (A^2 - a^2 + b^2) + b^2 A^2 = 2A^2 (A^2 - a^2 + b^2)$

$\Rightarrow$ $2A^4 + A^2(b^2 - 3a^2) + a^4 - a^2b^2 = 0$

$\Rightarrow$ $A^2 = \frac{(3a^2 - b^2) \pm \sqrt{(b^2 - 3a^2)^2 - 4 \cdot 2 \cdot (a^4 - a^2b^2)}}{4}$

$\Rightarrow$ $A^2 = \frac{(3a^2 - b^2) \pm (a^2 + b^2)}{4} = a^2$ or $\frac{a^2 - b^2}{2}$

$\because$ $A \neq a$

$\therefore$ $A^2 = \frac{a^2 - b^2}{2}$

$\therefore$ $B^2 = \frac{a^2 - b^2}{2}$

$\therefore$ $e = \sqrt{2} \Rightarrow e^2 = 2.$]

68. (B)

Sol. Let mid point of the chord is (h, k)

equation of chord of $x^2 - y^2 = a^2$ is

T = S,

$\Rightarrow hx - ky = h^2 - k^2$

$y = \frac{h}{k}x - \left(\frac{h^2 - k^2}{k}\right)$ .... (1)

(1) is tangent of $y^2 = 4ax$

condition of tangencys $c = \frac{a}{m}$

$\Rightarrow -\frac{h^2 - k^2}{k} = \frac{a}{\left(\frac{h}{k}\right)}$ $\Rightarrow -h^2 + k^2 = \frac{k^2 a}{h}$ $\Rightarrow -h^3 + hk^2 = k^2 a$ $\Rightarrow k^2 (h - a) = h^3$

Locus, $y^2 (x - a) = x^3$]

69. (D)

Sol. Let the focus be S(a, 0)

Since AS and BS are perpendicular to each other

$$\Rightarrow \frac{6}{12-a}\cdot\frac{2}{-(1+a)} = -1 \Rightarrow a = 0, 11$$

⇒ the focus is (0, 0) or (11, 0).The slope of the directrix be m. Also, distance of S from the line (y –2) = m(x + 1) is same as AS

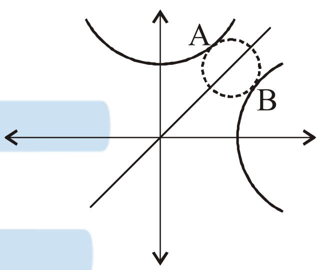
$\Rightarrow (m(13-6))^2 = (1+m^2)(180)$ or $(m(13)-6)^2 = (1+m^2)(37)$

⇒ 4 values of m are possible. Hence there are four parabolas.

70. (B)

Sol. Parabolas, $y = \frac{1}{2}(x^2+5)$ and $x = \frac{1}{2}(y^2+5)$ are symmetrical about the line y = x

∴ tangent at A is parallel to y = x



$$y = \frac{1}{2}(x^2+5)$$

$$\frac{dy}{dx} = \frac{1}{2}(2x) = 1 \Rightarrow x = 1$$

co-ordinate of A is (1, 3)

∴ co-ordinate of B is (3, 1)

Radius of circle (r) = $\frac{1}{2}(AB) = \frac{1}{2}\sqrt{4+4} = \sqrt{2}$

Area of circle = $\pi r^2 = 2\pi$.]

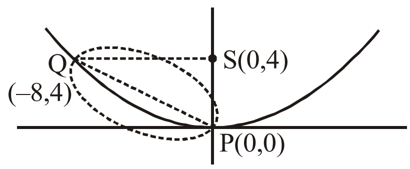
71. (C)

Sol. Given circle is x (x + 8) + y (y – 4) = 0

i.e. circle with (0, 0) and (– 8, 4) as diametric ends.

∵ (– 8, 4) is one end of latus rectum.

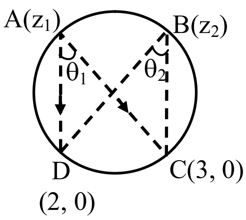
∴ ΔPQS is right angle



∴ PQ = circumdiameter = $\sqrt{8^2+4^2} = \sqrt{64+16} = 4\sqrt{5}$.]

72. (A)

Sol. $\arg\left(\frac{3-z_1}{2-z_1}\right) + \arg\left(\frac{2-z_2}{3-z_2}\right)$



$$= \arg\left(\frac{3-z_1}{2-z_1}\cdot\frac{2-z_2}{3-z_2}\right) = \theta_1 + \theta_2 = 0$$

Now if, $\left(\frac{3-z_1}{2-z_1}\cdot\frac{2-z_2}{3-z_2}\right)$ is a positive real number, then its argument will be zero. So chord DC subtends equal angle at A and B. So points are concyclic for k > 0.

73. (C)

Sol. $S = \sum_{n=0}^{100} (i)^{\lfloor n}$

$S = (i)^{\lfloor 0} + (i)^{\lfloor 1} + (i)^{\lfloor 2} + \ldots$

$= i + i - 1 + i^6 + i^{24} + (i)^{\lfloor 5} + (i)^{\lfloor 6} + \ldots\ldots + (i)^{\lfloor 100}$

$= 95 + 2i$

74. (C)

Sol. Let $u = \dfrac{z-1}{e^{\theta i}}$ $\qquad \Rightarrow \dfrac{e^{\theta i}}{z-1} = \dfrac{1}{4}$.

Now $\left(u + \dfrac{1}{u}\right) - \left(\bar{u} + \dfrac{1}{\bar{u}}\right) = 0$

$\Rightarrow (u - \bar{u})\left(1 - \dfrac{1}{u\bar{u}}\right) = 0$

If u is not purely real, then $u\bar{u} = 1$

$\Rightarrow \left|\dfrac{z-1}{e^{\theta i}}\right| = 1 \Rightarrow |z-1| = 1$

75. (D)

Sol. $z = \dfrac{\alpha + \beta t}{\gamma + \delta t} \Rightarrow (\gamma + \delta t)\, z = \alpha + \beta t$

$\Rightarrow (\delta z - \beta)t = \alpha - \gamma z$

$\Rightarrow \qquad t = \dfrac{\alpha - \gamma z}{\delta z - \beta} \qquad [\because \alpha\delta - \beta\gamma \neq 0\,]$

As t is real, $\dfrac{\alpha - \gamma z}{\delta z - \beta} = \dfrac{\bar{\alpha} - \overline{\gamma z}}{\bar{\delta}\bar{z} - \bar{\beta}}$

$\Rightarrow \qquad (\alpha - \gamma z)(\bar{\delta}\bar{z} - \bar{\beta}) = (\bar{\alpha} - \bar{\gamma}\bar{z})(\delta z - \beta)$

$\Rightarrow (\bar{\gamma}\delta - \gamma\bar{\delta})z\bar{z} + (\gamma\bar{\beta} - \bar{\alpha}\delta)z + (\alpha\bar{\delta} - \beta\bar{\gamma})\bar{z}$

$= (\alpha\bar{\beta} - \bar{\alpha}\beta) \qquad \ldots.(1)$

Since $\dfrac{\gamma}{\delta}$ is real, $\dfrac{\gamma}{\delta} = \dfrac{\gamma}{\delta}$ or $\gamma\bar{\delta} - \delta\bar{\gamma} = 0$

Therefore (1) can be written as $\bar{a}z + a\bar{z} = c \quad \ldots(2)$

where $a = i(\alpha\bar{\delta} - \beta\bar{\gamma})$ and $c = i(\bar{\alpha}\beta - \alpha\bar{\beta})$

Note that $a \neq 0$ for if $a = 0$ then

$a\bar{\delta} - \beta\bar{\gamma} = 0 \Rightarrow \dfrac{\alpha}{\beta} = \dfrac{\bar{\gamma}}{\bar{\delta}} = \dfrac{\gamma}{\delta} \quad [\because \dfrac{\gamma}{\delta}$ is real$]$

$\Rightarrow \qquad \alpha\delta - \beta\gamma = 0,$

which is against hypothesis.

Also, note that $c = i(\bar{\alpha}\beta - \alpha\bar{\beta})$ is a purely real number.

Thus, $z = \dfrac{\alpha + \beta t}{\gamma + \delta t}$ represents a straight line.

76. (A)

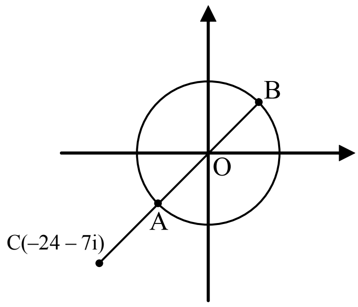
Sol. Note that $|z| = 6$ represents a circle. As $|z_2| = 6$,

$|z_1 + z_2| = |z_2 - (-24 - 7i)|$

represent distance between a point on the circle $|z| = 6$ and the point $C(-24 - 7i)$.

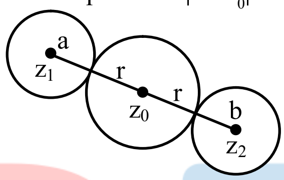
$|z_1 + z_2|$ will be greatest and least at points

B and A which are the end points of the diameter of the cirlcle through C. As $OC = 25$, $CA = OC - OA = 25 - 6 = 19$ and $CB = OC + OB = 25 + 6 = 31$.



77. (B)

Sol. Centre of the circle be z_0 and radius r. Then its equation is $|z - z_0| = r$ circle (A) touches the circle $|z - z_1| = a$ externally.



Distance between centres = sum of radii

$|z_0 - z_1| = a + r$ ...(1)

$|z_0 - z_2| = b + r$ ...(2)

(1) – (2)

$|z_0 - z_1| - |z_0 - z_2| = a - b$

$\therefore z_0$ lies on the curve $|z - z_1| - |z - z_2| = a - b$ which is equation of a hyperbola.

78. (B)

Sol. $\arg(zw) = \pi \Rightarrow zw = z.\bar{w} \Rightarrow zw.w = \bar{z}\,\bar{w}.w$

$\Rightarrow z.w^2 = \bar{z}.|w|^2 \Rightarrow zw^2 = \bar{z}\,|z|^2$

$\Rightarrow zw^2 = \bar{z}z\bar{z}$

$\Rightarrow w^2 = (\bar{z})^2$

$\Rightarrow \bar{z} = w \text{ or } -w$

$\Rightarrow z = \bar{w} \text{ or } -\bar{w}$

but only $z = -\bar{w}$ satisfies $\arg(zw) = \pi$

79. (C)

Sol. $\dfrac{1-ix}{1+ix} = a - ib \Rightarrow 1 - ix = (a - ib)(1 + ix)$

$\Rightarrow 1 - ix = a + aix - ib + bx$

$\Rightarrow (1 - a + ib) = x\,[ai + b + i]$

$$\Rightarrow x = \frac{(1-a)+ib}{b+i(a+1)} \times \frac{b-i(a+1)}{b-i(a+1)} = \frac{b(1-a)+b(a+1)+i(b^2-(1-a^2))}{b^2+(a+1)^2}$$

x is real if $b^2 + a^2 - 1 = 0$

i.e. $a^2 + b^2 = 1$ $\therefore$ [C] is correct.

80. (C)

Sol. $\left|Z+\frac{1}{Z}\right| = \left|Z-\left(-\frac{1}{Z}\right)\right| \geq |Z| \sim \left|-\frac{1}{Z}\right| \geq 3 \sim \frac{1}{3} = \frac{8}{3}$

81. 136

Sol. It is to be noted that two of the numbers need to be conjugates and one number must be real, as the coefficients of the cubic are all real.

Three roots are, $\omega+3i$, $\omega+9i$ and $2\omega-4$

let $\omega+3i$ is real, hence $\omega=\alpha-3i$ where $\alpha \in R$

then $(\alpha-3i+9i)$ and $(2\alpha-6i-4)$

i.e. $\alpha+6i$ and $(2\alpha-4)-6i$ must be complex conjugate $\Rightarrow \alpha=2\alpha-4$

$\therefore$ $\alpha=4$

Hence the roots are

$4, 4+6i, 4-6i$ (other options not possible)

the equation is

$(z-4)(z^2-8z+5z) \Rightarrow z^3-12z^2+84z-208$

Hence $|a+b+c|=|-12+84-208|=136$. **Ans.**]

82. 100

Sol. Let $S=1+\frac{2}{z}+\frac{3}{z^2}+\frac{4}{z^3}+.......$

$$\frac{S}{z} = +\frac{1}{z}+\frac{2}{z^2}+\frac{3}{z^3}+......$$

$$- \quad - \quad - \quad - \quad -$$

$$S\left(1-\frac{1}{z}\right)=1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+......$$

$$\frac{S(z-1)}{z}=\frac{1}{1-\frac{1}{z}}=\frac{z}{z-1}$$

$$S=\frac{z^2}{(z-1)^2}$$

$\therefore$ $i z^2=\frac{z^2}{(z-1)^2}$ $(z\neq 0)$

$i=\frac{1}{(z-1)^2} \Rightarrow (z-1)^2=\frac{1}{i}=-i \Rightarrow z-1=\pm\sqrt{-i}$

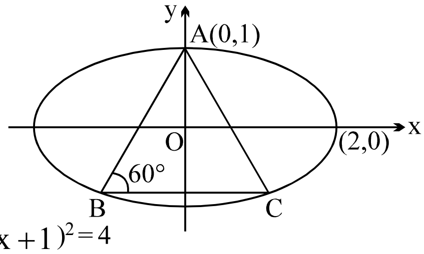
$z=1\pm\sqrt{-i}$

given $z=n\pm\sqrt{-i}$

$\Rightarrow$ $100n=100$ Ans.]

83. 937

Sol. $\frac{x^2}{4}+y^2=1$ ....(1)



equation of the side AB

$y-1=\sqrt{3}\,(x-0) \Rightarrow$ $y=\sqrt{3}\,x+1$ ....(2)

solving (1) and (2)

$$\frac{x^2}{4}+(\sqrt{3}x+1)^2=1 \Rightarrow x^2+4(\sqrt{3}x+1)^2=4$$

$$x^2+4(3x^2+1+2\sqrt{3}\,x)=4 \Rightarrow 13x^2+8\sqrt{3}\,x=0$$

$$x=0 \quad \text{or} \quad x=-\frac{8\sqrt{3}}{13} \text{ in (2)}$$

$$y=-\sqrt{3}\left(\frac{8\sqrt{3}}{13}\right)+1=1-\frac{24}{13}=-\frac{11}{13}; \quad \text{Hence} \quad B=\left(-\frac{8\sqrt{3}}{13},-\frac{11}{13}\right)$$

$$(AB)^2=\left(\frac{8\sqrt{3}}{13}\right)^3+\left(\frac{24}{13}\right)^2=\frac{64\cdot 3+24^2}{169}=\frac{192+576}{169}=\frac{768}{169} \Rightarrow AB=\sqrt{\frac{768}{169}}$$

$\Rightarrow$ $m+n=768+169=937$ Ans.]

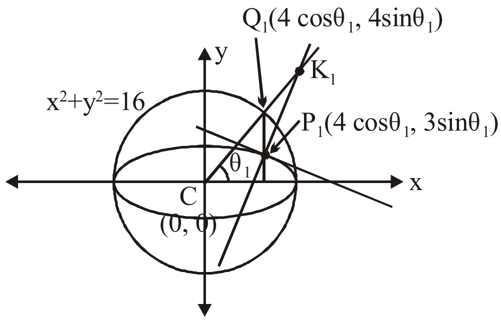
84. 8

Sol. Equation of normal at $P_1\,(4\cos\theta_1,\ 3\sin\theta_1)$ is

$$\frac{4x}{\cos\theta_1}-\frac{3y}{\sin\theta_1}=7 \quad ...(1)$$

Also, equation of CQ_1 is

$$y=\left(\frac{\sin\theta_1}{\cos\theta_1}\right)x \quad ...(2)$$



$\therefore$ Solving (1) and (2), we get

$$\frac{4x}{\cos\theta_1}-\frac{3}{\sin\theta_1}\left(\frac{\sin\theta_1}{\cos\theta_1}\right)x=7$$

$\Rightarrow$ $\frac{x}{\cos\theta_1}=7$ $\Rightarrow$ $x=7\cos\theta_1,\ y=7\sin\theta_1$

So, $K_1=(7\cos\theta_1, 7\sin\theta_1)$ $\Rightarrow$ $CK_1=7$

Similarly, $CK_2=CK_3=.....=CK_n=7$

$\therefore$ $\sum_{i=1}^{n} CK_i=56$ $\Rightarrow 7n=56$ $\Rightarrow$ $n=\frac{56}{7}=8.$ **Ans.**]

85. 10

Sol. Let $E: \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $H: \frac{x^2}{A^2}-\frac{y^2}{B^2}=1$

$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{4}$; $e_E{}^2 = 1-\frac{b^2}{a^2} \Rightarrow \frac{b^2}{a^2}=\frac{3}{4}$

So, $a^2 = \frac{28}{3}$ and $b^2 = 7$

$\Rightarrow E: \frac{x^2}{\frac{28}{3}}+\frac{y^2}{7}=1$

Now, slope of tangent at M (2, 2) on ellipse = $\frac{-3}{4}$

So, slope of tangent at M (2, 2) on hyperbola $\Rightarrow \frac{B^2}{A^2}=\frac{4}{3}$

As, $e_h{}^2 = 1+\frac{B^2}{A^2} \Rightarrow e_h{}^2 = 1+\frac{4}{3}=\frac{7}{3}$

So, $e_h = \sqrt{\frac{7}{3}}$. [**Note** that ellipse and hyperbola are confocal.].

86. 0

Sol. Let CP and CD be a pair of conjugate diameters of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. Then the co- ordinates of P and D are (a sec θ, b tan θ) and (a tan θ, b sec θ) respectively.

It is given that the line $lx + my + n = 0$ meets the hyperbola at P and D. Therefore,

$al \sec\theta + bm \tan\theta + n = 0$

and $al \tan\theta + bm \sec\theta + n = 0$

$\Rightarrow al \sec\theta + bm \tan\theta + n = -n$

and $al \tan\theta + bm \sec\theta = -n$

$\Rightarrow (al \sec\theta + bm \tan\theta)^2 = n^2$ … (i)

And $(al \tan\theta + bm \sec\theta)^2 = n^2$ … (ii)

on subtracting Equation (ii) from (i), we get

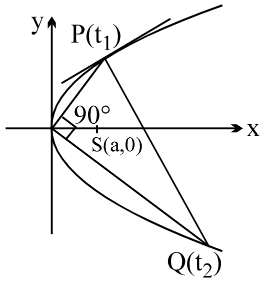
$a^2l^2 (\sec^2\theta - \tan^2\theta) + b^2m^2 (\tan^2\theta - \sec^2\theta) = 0$

$\Rightarrow a^2l^2 - b^2m^2 = 0$

87. 3

Sol. $\frac{2}{t_1}\cdot\frac{2}{t_2}=-1 \Rightarrow t_1 t_2=-4$; also $t_2=-t_1-\frac{2}{t_1}$ ....(1)

$t_1t_2=-t_1^2-2 \Rightarrow -4+2=-t_1^2 \Rightarrow t_1^2=2$



also $t_2^2=t_1^2+\frac{4}{t_1^2}+4$ squaring (1)

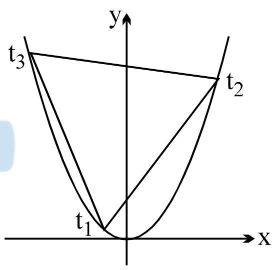
$t_2^2=2+2+4=8$

Now $SQ=a(1+t_2^2)=a(1+8)=9a$ and $SP=a(1+t_1^2)=a(1+2)=3a$

$\therefore \frac{SQ}{SP}=3 \Rightarrow SQ=3SP \Rightarrow \lambda=3$]

88. 14

Sol. Say $m_1=2$



$m_1=\frac{t_2^2-t_1^2}{t_2-t_1}=t_1+t_2=2$ .....(1)

now $\frac{2-m_2}{1+2m_2}=\sqrt{3}$

$2-m_2=\sqrt{3}+2\sqrt{3}\,m_2$

$m_2(2\sqrt{3}+1)=2-\sqrt{3}$

$t_1+t_3=m_2=\frac{2-\sqrt{3}}{1+2\sqrt{3}}$ ....(2)

again $\frac{m_3-2}{1+2m_3}=\sqrt{3}$

$m_3-2=\sqrt{3}+2\sqrt{3}\,m_3$

$m_3(1-2\sqrt{3})=2+\sqrt{3} \Rightarrow m_3=\frac{2+\sqrt{3}}{1-2\sqrt{3}}=t_2+t_3$

hence $2(t_1+t_2+t_3)=2+\frac{2-\sqrt{3}}{1+2\sqrt{3}}+\frac{2+\sqrt{3}}{1-2\sqrt{3}}$

as $t_1+t_2+t_3=\frac{p}{q}$

hence $\frac{2p}{q}=2+\frac{2-\sqrt{3}}{2\sqrt{3}+1}-\frac{2+\sqrt{3}}{2\sqrt{3}-1}=2+\frac{(2-\sqrt{3})(2\sqrt{3}-1)-(2+\sqrt{3})(2\sqrt{3}+1)}{11}$

$=2+\frac{(5\sqrt{3}-8)-(5\sqrt{3}+8)}{11}$

$\frac{2p}{q}=2-\frac{16}{11}=\frac{6}{11} \Rightarrow \frac{p}{q}=\frac{3}{11} \Rightarrow p+q=14$ Ans.

89. 63

Sol. $z^5 + z^4 + z^3 + z^2 + z + 1 = 0$

$(z-\alpha_1)(z-\alpha_2)(z-\alpha_3)(z-\alpha_4)(z-\alpha_5)$

$= z^5+z^4+z^3+z^2+z+1$

Putting $z = 2$

$\Rightarrow (2-\alpha_1)(2-\alpha_2)(2-\alpha_3)(2-\alpha_4)(2-\alpha_5)$

$= 32+16+8+4+2+1$

$$\therefore \prod_{i=1}^{5}(2-\alpha_i) = 63$$

90. 6

Sol. Equation of the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

The point P $(-(a-2), 6)$ lies on it

$$\frac{(a-2)^2}{a^2} + \frac{36}{b^2} = 1$$

$$\frac{36}{b^2} = 1 - \frac{(a-2)^2}{a^2} \Rightarrow \frac{4(a-1)}{a^2}$$

$$\frac{9}{b^2} = \frac{a-1}{a^2}$$

$$b^2 = \frac{9a^2}{a-1} \Rightarrow 9\left\{a+1+\frac{1}{a-1}\right\}$$

$$\frac{d(b^2)}{da} = 9\left\{1-\frac{1}{(a-1)^2}\right\} \text{ and } \frac{d^2b^2}{da^2} = \frac{18}{(a-1)^3}$$

For extreme value of b $\quad \frac{db^2}{da} = 0 \Rightarrow a = 2$

$$b^2 = \frac{9(2)^2}{2-1} \Rightarrow b = 6\text{m}$$

greatest height of the arch is 6m.