JEE MAIN ANSWER KEY & SOLUTION

PAPER CODE :- PART TEST-5
CLASS-XII

ANSWER KEY

PHYSICS

1.	(D)	**2.**	(A)	**3.**	(A)	**4.**	(C)	**5.**	(A)	**6.**	(B)	**7.**	(A)
8.	(C)	**9.**	(B)	**10.**	(D)	**11.**	(D)	**12.**	(D)	**13.**	(C)	**14.**	(A)
15.	(B)	**16.**	(D)	**17.**	(C)	**18.**	(B)	**19.**	(A)	**20.**	(B)	**21.**	11
22.	24	**23.**	5	**24.**	2	**25.**	81	**26.**	3	**27.**	100	**28.**	7
29.	200	**30.**	2										

CHEMISTRY

31.	(A)	**32.**	(D)	**33.**	(C)	**34.**	(B)	**35.**	(D)	**36.**	(C)	**37.**	(B)
38.	(B)	**39.**	(D)	**40.**	(B)	**41.**	(B)	**42.**	(B)	**43.**	(D)	**44.**	(A)
45.	(D)	**46.**	(D)	**47.**	(A)	**48.**	(D)	**49.**	(A)	**50.**	(B)	**51.**	18
52.	2	**53.**	0	**54.**	63	**55.**	4	**56.**	3	**57.**	99	**58.**	5
59.	197	**60.**	6										

MATHEMATICS

61.	(C)	**62.**	(C)	**63.**	(A)	**64.**	(C)	**65.**	(C)	**66.**	(A)	**67.**	(A)
68.	(D)	**69.**	(D)	**70.**	(D)	**71.**	(D)	**72.**	(D)	**73.**	(B)	**74.**	(D)
75.	(D)	**76.**	(C)	**77.**	(C)	**78.**	(D)	**79.**	(C)	**80.**	(B)	**81.**	4
82.	8	**83.**	3	**84.**	600	**85.**	1	**86.**	24	**87.**	5	**88.**	3
89	1	**90.**	1										

SOLUTIONS

PHYSICS

1. (D)

2. (A)

Sol. In given situation output C is high only when both inputs A and B are high so logic circuit gate is AND gate

3. (A)

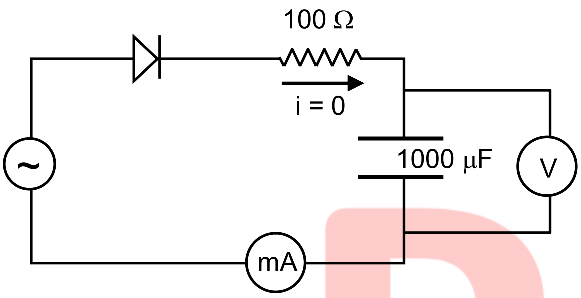
Sol. Shortest wavelength of Bracket series corresponds to the transition of electron $n_1 = 4$ and $n_2 = \infty$ and the shortest wavelength of Balmer series corresponds to the transition of electron between $n_1 = 2$ and $n_2 = \infty$. So

$$(Z^2)\left(\frac{13.6}{16}\right) = \left(\frac{13.6}{4}\right)$$

$\therefore$ $Z^2 = 4$ or $Z = 2$

4. (C)

Sol.



In this state the circuit will work as half wave rectifier and the voltage across capacitor will be equal to average value of voltage of ac source over a half cycle.

5. (A)

Sol. Let n be the average no. of electrons per unit volume, present in the beam of electrons, then impulse-momentum theorem gives

$(nAv)(mv) = F$, $P = F/A$

$\Rightarrow$ $n = P/(mv^2)$

$\therefore$ $i = nAve = \dfrac{APe}{mv}$

6. (B)

Sol. $t = \dfrac{2r}{v} = \dfrac{210^{-15}\,m}{3\times10^{7}\,m/s} = \dfrac{2}{3}\times10^{-22}\,s$

7. (A)

Sol. $evB = \dfrac{mv^2}{R} \Rightarrow v = \dfrac{e}{m}BR$

K.E. of photoelectrons

$K = \dfrac{1}{2}mv^2 = \dfrac{e^2B^2R^2}{2m} = 2.97\times10^{-15}\,J$

$K.E. = 2.97\times10^{-15}\,J = 18.6\,KeV$

$K.E. = E_P - E(K)$

$E(K) = E_P - K.E. = 24.8 - 18.6 = 6.2\,K.E.$

8. (C)

Sol. $I = n_e A v_d$

$$\frac{I_e}{I_h} = \frac{n_e \times (v_d)_e}{n_h \times (v_d)_h}$$

Here, $\frac{n_e}{n_h} = \frac{7}{5}, \frac{I_e}{I_h} = \frac{7}{4}$

$$\frac{7}{4} = \frac{7}{5} \times \frac{(v_d)_e}{(v_d)_h} \Rightarrow \frac{(v_d)_e}{(v_d)_h} = \frac{5}{7} \times \frac{7}{4} = \frac{5}{4}.$$

9. (B)

Sol. $\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = ev$ ...(i)

$\frac{hc}{2\lambda} - \frac{hc}{\lambda_0} = e\frac{v}{3}$ ...(ii)

Multiplying equation (ii) by (iii) and subtract from equation (i)

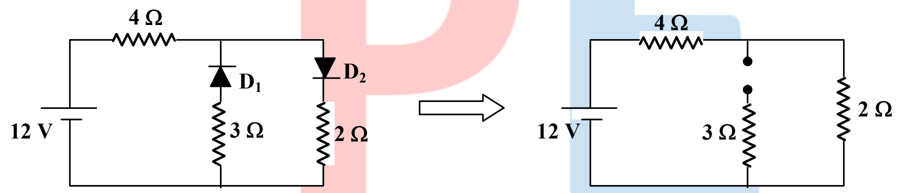
$$\frac{hc}{\lambda} - \frac{hc}{\lambda_0} - \frac{3hc}{2\lambda} + \frac{3hc}{\lambda_0} = 0$$

$$\frac{2hc}{\lambda_0} = \frac{hc}{2\lambda}$$

$\lambda_0 = 4\lambda$

10. (D)

Sol



amp ∴ (D) is correct.

11. (D)

Sol. $\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \phi$

$$v = \sqrt{\frac{2}{m}\left(\frac{hc}{\lambda} - \phi\right)}$$

$$v' = \sqrt{\frac{2}{m}\left(\frac{4hc}{3\lambda} - \phi\right)} > \sqrt{\frac{4}{3}}\,v$$

12. (D)

Sol. $4 \to 3 \Rightarrow$ emitted energy = 0.65 eV

$\lambda_{U.V.} < \lambda_{I.R.}$

$E_{U.V.} > E_{I.R.}$

(A) when electron transit $2 \to 1$ emitted energy is $10.2Z^2$eV

(B) $3 \to 2 \Rightarrow 1.92\ Z^2$ eV

(C) $4 \to 2 \Rightarrow 2.55\ Z^2$eV

(D) $5 \to 4 \Rightarrow$ less than $0.65Z^2$eV

So option (D) is correct.

13. (C)

Sol. (A) Franck – Hertz Experiment is associated with Discrete energy levels of atom

(B) Photo electric experiment is associated with particle nature of light and Davisson–Germer experiment is associated with wave nature of electron.

14. (A)

Sol. Using $\frac{1}{f_{carrier}} << RC$

We get time constant $RC = 1000 \times 10^{-12} = 10^{-9}$s

Now $v = \frac{1}{T} = \frac{1}{10^{-9}} = 10^{9}$ Hz

Thus, the value of carrier frequency should be much less than 10^9 Hz, say 100 kHz.

$\Rightarrow \frac{v_m}{v_s} = \sqrt{\frac{L_s}{L_m}} \Rightarrow L_s < L_m$

15. (B)

Sol. For less than 0.5 force is positive so it is repulsive in nature

For greater than 10 fermi it is almost zero

16. (D)

17. (C)

Sol. $\frac{1}{2} mv^2_{max} = h\nu - h\nu_o$

$= h(\nu - \nu_o)$

This is Einstein's equation of photoelectric effect.

18. (B)

Sol. $Y = \overline{\overline{A}.\overline{B}} = A + B$ i.e. OR gate

19. (A)

Sol. Upper diode is in forward bias,

So, $i = V/R = 2V/20\Omega = 0.1$ A

20. (B)

Sol. $P = VI \Rightarrow I = \frac{P}{V} = \frac{100}{0.5} \times 10^{-3}$, $\quad I = \frac{1}{5}$

$I = \frac{V_{net}}{R_{net}} = \frac{1.5 - 0.5}{R} = \frac{1}{R}$

$R = 5$

21. 11

Sol. For silicon diode barrior potential is 0.7V

so $I = \frac{3 - 0.7}{200}$

$= 0.0115$A

$= 11.5$mA

22. 24

Sol. In the reaction of fusion,

$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{4}_{2}He + Q$

Energy of reactants is 4×1.15 for 2 deuterons

$= 4.60$ MeV

Energy of the product $= 4 \times 7.1 = 28.4$ MeV

$\therefore$ Q of reaction $= 28.4 - 4.6 = 23.8$ MeV

23. 5

24. 2

Sol.

$$eV_1 = \frac{hc}{\lambda_1} - \phi$$

$$eV_2 = \frac{hc}{\lambda_2} - \phi$$

$$e(V_2 - V_1) = hc\left(\frac{\lambda_1 - \lambda_2}{\lambda_1\lambda_2}\right)$$

$$V_2 - V_1 = \frac{hc}{e}\left(\frac{\lambda_1 - \lambda_2}{\lambda_1\lambda_2}\right)$$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}} \times \frac{100}{6\times10^{-5}}$$

$$= \frac{66}{32} \times 10^{-34+8+2+19+5}$$

$$= \frac{33}{16} = 2.0625 \text{ volt} \approx 2 \text{ volt}$$

25. 81

Sol. For full wave rectifier $\eta = \dfrac{81.2}{1+\dfrac{r_f}{R_L}} \Rightarrow \quad n_{max} = 81.2\% \quad (r_f << R_L)$

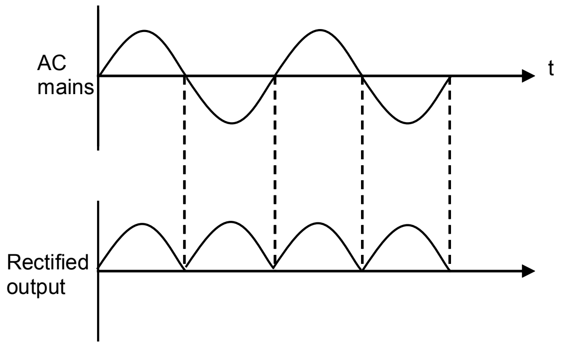
26. 3

Sol. $E_n = -\dfrac{13.6 \text{ eV}}{n^2} = -1.51$ eV

$n = 3 \quad L = 3\left(\dfrac{h}{2\pi}\right)$

27. 100

Sol. frequency doubles = $100H_2$



28. 7

Sol. In fig. germanium diode is reverse biased and silicon diode is forward biased. Therefore, there will be no current in the branch of germanium diode. The potential barrier of silicon diode is 0.7V. Therefore, for conduction minimum potential difference across silicon is 0.7 V.

29. 200

Sol. The basic equation of diode-circuit is

$$RI + V_0 = V_B$$

$$\Rightarrow \quad R = \frac{V_B - V_0}{I}$$

Here $V_B = 1.5V, V_0 = 0.3V, I = 5mA = 6 \times 10^{-3}A$

$$\therefore R = \frac{1.5 - 0.3}{6 \times 10^{-3}} = \frac{1.2 \times 10^{3}}{6} = 0.2 \times 103\Omega = 200\Omega$$

30. 2

Sol. Energy released = increase in BE = 6.5 × 120 + 7.0 × 80 – 6 × 203 = 122 MeV.

31. (A)
Sol. Glucose is monosacchoride.

32. (D)
Sol. Secondary structure of protein is regular folding pattern of continuous portion of the polypeptide chain

33. (C)
Sol. Amino acid are linked through peptide linkage in protein structure.

34. (B)
Sol. Glycine is simplest amino acid because R = H atom in amino acid structure.

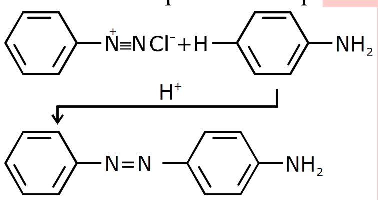
35. (D)
Sol. End product of protein digestion is α–amino acid because Amino acids are structural unit of protein.

36. (C)
Sol. Calorific value order is fat > carbohydrate > protein

37. (B)
Sol. Riboflavin deficiency causes pellagra.

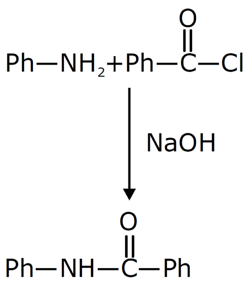
38. (B)
Sol. Cod liver oil is good source of vitamin A & D.

39. (D)
Sol. This is an example of electrophilic substitution reaction [coupling reaction]

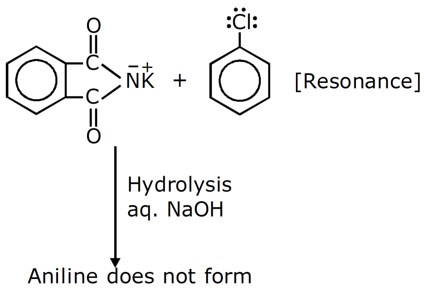


p-Aminoazobenzene
(yellow dye)
$+Cl^- + H_2O$

40. (B)
Sol. Schotten - Baumenn reaction



41. (B)
Sol.



42. (B)

Sol. $R-NH_2 + CHCl_3 + KOH \rightarrow R-NC + 3KCl + 3H_2O$

1^O – Amine   Alkyl isocyanide

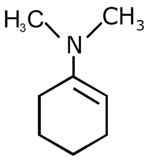
This is carbyl amine reaction

43. (D)

Sol. Wurtz reaction is used for the formation of alkane.

44. (A)

Sol.

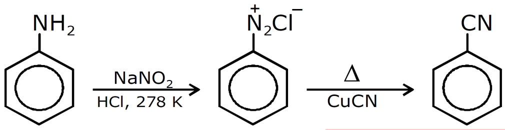
It forms enamine 

45. (D)

Sol. $CH_3CH_2NH_2 + CHCl_3 + 3KOH \rightarrow C_2H_5NC + 3KCl + 3H_2O$

46. (D)

Sol.



47. (A)

Sol. Methyl isocyanate is poisonous gas.

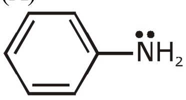
48. (D)

Sol. $R-NH_2 + CHCl_3 + KOH \xrightarrow{\Delta} R-NC$

1^O – amine   Chloroform   Alkyl isocyanide

It is carbylamine reaction or isocyanide test.

49. (A)

Sol. 

Delocalised lone pair electrons.

(least Basic)

Order of basic strength in aqueous solution

$\Rightarrow 2^o > 1^o > 3^o$ amine > aniline.

Basic strength $\propto K_b \propto \frac{1}{pK_b}$

$(CH_3)_2NH$ is most basic so it has smallest pKb.

50. (B)

Sol. In Kjeldahl method, nitrogen containing compounds when react with H_2SO_4, then $(NH_4)_2SO_4$ is formed which means NH_3 must be released in the reaction. Only in (III) option NH_3 can be released.

51. 18

Sol. Per CO_2 require 3 molecule of ATP.

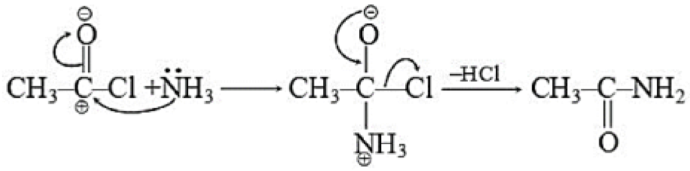
1 molecule of Glucose gives 6 CO_2 so Ans is 18 ATP.

52. 2

Sol. 2^{nd}

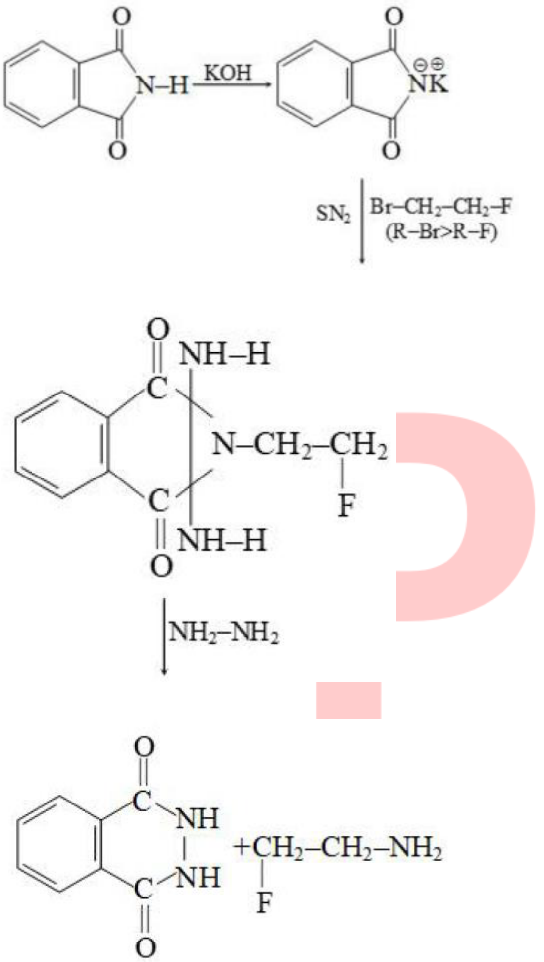
53. 0

Sol.



54. 63

Sol.



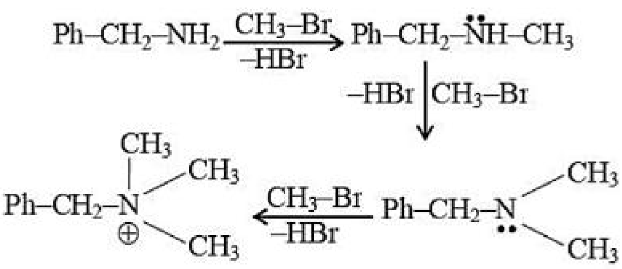
55. 4

Sol. Balanced reaction is

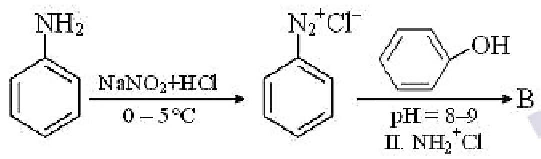
$$R-\overset{O}{\overset{||}{C}}-NH_2 + 4NaOH + Br_2 .. R-NH_2 + 2NaBr + Na_2CO_3 + H_2O.$$

56. 3

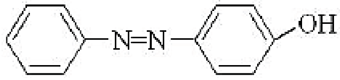
Sol.



57. 99



Sol.



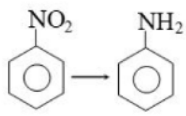
(B)

Molar mass of B = 198 = x

$$\frac{x}{2} = \frac{198}{2} = 99.00$$

58. 5

Sol.

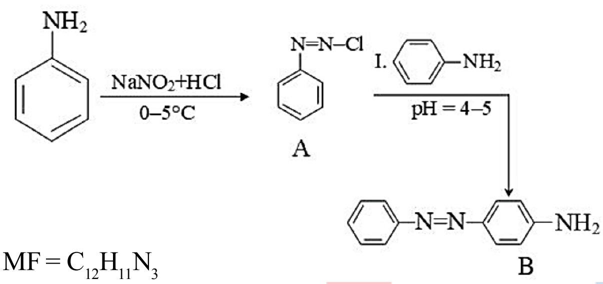


Reagents used can be

(i) Sn + HCl (ii) Fe + HCl (iii) Zn + HCl (iv) H_2 - Pd (v) H_2 (Raney Ni)

59. 197

Sol.

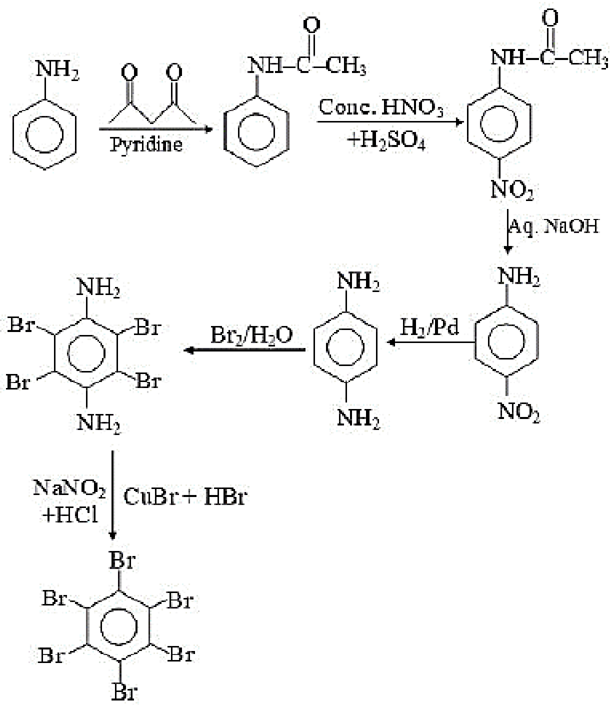


MF = $C_{12}H_{11}N_3$

$12 \times 12 \times 11 \times 1 + 14 \times 3 = 197$

60. 6

Sol.



61. (C)

Sol. Let $\vec{b} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$\vec{a} \times \vec{b} = \hat{j} - \hat{k}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ \alpha & \beta & \gamma \end{vmatrix} = \hat{j} - \hat{k}$$

$\Rightarrow \beta - \gamma = 0, \alpha - \gamma = 1, \alpha - \beta = 1$

$\Rightarrow \beta = \gamma, \alpha = 1 + \gamma, \alpha = 1 + \beta$

$\vec{a} \cdot \vec{b} = 1 \Rightarrow \alpha + \beta + \gamma = 1$

$\Rightarrow \beta + 1 + \beta + \beta = 1 \Rightarrow \beta = 0$

$\therefore \alpha = 1, \gamma = 0$

$\therefore \vec{b} = \hat{i}$.

62. (C)

Sol. A vector bisecting the angle between $\bar{a}$ and is

$\frac{\bar{a}}{|\bar{a}|} \pm \frac{\bar{b}}{|\bar{b}|}$; in this case $\frac{2i + j - k}{\sqrt{6}} \pm \frac{i - 2j + k}{\sqrt{6}}$

(i.e.) $\frac{3i - j}{\sqrt{6}}$ or $\frac{i + 3j - 2k}{\sqrt{6}}$

A vector of magnitude 3 along these vectors is $\frac{3(3i - j)}{\sqrt{10}}$ or $\frac{3(i + 3j - 2k)}{\sqrt{14}}$

Now, $\frac{3}{\sqrt{14}}(i + 3j - 2k).(i - 2j + k)$ is negative and hence

$\frac{3}{\sqrt{14}}(i + 3j - 2k)$ makes an obtuse angle with

63. (A)

Sol. $\vec{a} = (1, 3, \sin 2\alpha)$ makes an obtuse angle with z-axis

$\sin 2\alpha < 0$

$\vec{b}$ and $\vec{c}$ are orthogonal $\vec{b} \cdot \vec{c} = 0$

$\Rightarrow \tan^2\alpha - \tan\alpha - 6 = 0$

$\Rightarrow \tan\alpha = 3$ or -2

If $\tan\alpha = 3$

$\sin 2\alpha = \frac{2\tan\alpha}{1 + \tan^2\alpha} = \frac{3}{5} > 0$ (not possible),

$\tan\alpha = -2$

$\tan 2\alpha = \frac{4}{3} > 0$

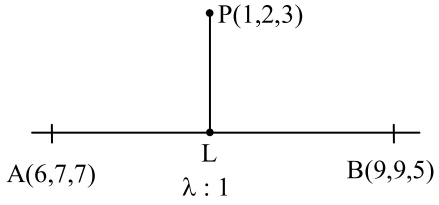
$\sin 2\alpha < 0$

2α lies in the third quadrant $\Rightarrow \frac{\alpha}{2}$ lies in 2^{nd} quadrant $\therefore \sqrt{\sin\alpha/2}$ is valid and

$\alpha = (4n + 1)\pi - \tan^{-1} 2$.

64. (C)

Sol.



$L \equiv \left(\frac{9\lambda+6}{\lambda+1}, \frac{9\lambda+7}{\lambda+1}, \frac{5\lambda+7}{\lambda+1}\right)$

$PL \perp AB$

65. (C)

Sol.

$$\left.\begin{aligned} 2\vec{a}+\vec{b} &= (1,\ 1,\ 1) \\ \vec{a}+2\vec{b} &= (1,\ 1,\ -1) \end{aligned}\right\}$$

$\Rightarrow 3\vec{a} = (1, 1, 3)$ and $3\vec{b} = (1, 1, -3)$

$\vec{a} = \left(\frac{1}{3}, \frac{1}{3}, 1\right)$ and $\vec{b} = \left(\frac{1}{3}, \frac{1}{3}, -1\right)$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{\frac{1}{9}+\frac{1}{9}-1}{\sqrt{\frac{11}{9}}\sqrt{\frac{11}{9}}} = -\frac{7}{11}$$

$$\Rightarrow \theta = \cos^{-1}\left(-\frac{7}{11}\right)$$

66. (A)

Sol. Let the point of intersection be $(\lambda,\lambda,\lambda)$

$\therefore \sin A + \sin B + \sin C = 2(\sin 2A + \sin 2B + \sin 2C)$

$\therefore 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} = 8\sin A\sin B\sin C$

$\therefore \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = \frac{1}{16}$.

67. (A)

Sol. Given,

$\vec{u}+\vec{v}+\vec{w} = \vec{0}$

If $|\vec{u}| = 3$; $|\vec{v}| = 4$ and $|\vec{w}| = 5$

Means $\vec{u}$, $\vec{v}$ and $\vec{w}$ form a right angled triangle, right angled at the angle between $\vec{u}$ and $\vec{v}$

$\therefore \vec{u}\cdot\vec{v} = 0$;

Further $\vec{u}\cdot\vec{w} = \vec{u}\ |\vec{w}|\cos\theta$,

but, $\cos\theta = -\frac{3}{5}$

$\vec{u}\cdot\vec{w} = (3)(5)\left(-\frac{3}{5}\right) = -9$

$\vec{v}\cdot\vec{w}\ \vec{v} = |\vec{w}|\cos\phi$

but $\cos\phi = -\frac{4}{5}$

$\vec{v}\cdot\vec{w} = (4)(5)\left(-\frac{4}{5}\right) = -16$

$\Rightarrow \vec{u}\cdot\vec{v}+\vec{v}\cdot\vec{w}+\vec{u}\cdot\vec{w} = 0-16-9 = -25$

68. (D)

Sol. Here,

$$\frac{x-1/3}{1/6}=\frac{y+1/3}{1/3}=\frac{z-1}{1/2} \qquad \ldots(i)$$

Line $\vec{r}=\vec{a}+\vec{b}\lambda$ can be written as

$$\frac{x-a_1}{b_1}=\frac{y-a_2}{b_2}=\frac{z-a_3}{b_3}$$

Where $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

and $\vec{b}=b_1\hat{i}+b_2\hat{j}+a_3\hat{k}$

so the equation (i) can be written as :

$$\vec{r}\left(\frac{1}{3}\hat{i}-\frac{1}{3}\hat{j}+\hat{k}\right)+\lambda(\hat{i}+2\hat{j}+3\hat{k}).$$

69. (D)

Sol. Shortest distance between B and plane OAC

$$(h)=\frac{\overrightarrow{OA}\times\overrightarrow{OC}.\ \overrightarrow{OB}}{|\overrightarrow{OA}\times\overrightarrow{OC}\ |}$$

here $\overrightarrow{OA}\times\overrightarrow{OC}\ \overrightarrow{OB}.=\begin{vmatrix}1 & 2 & 3\\ 2 & -1 & 1\\ 3 & 1 & 2\end{vmatrix}=1(-2-1)+2(3-4)+3\,(2+3)=10$

$\overrightarrow{OA}\times\overrightarrow{OC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 3 & 1 & 2\end{vmatrix}=\hat{i}\,(-2-1)+\hat{j}\,(3-4)+\hat{k}\,(2+3)=-3\hat{i}-\hat{j}+5\hat{k}$

$|\overrightarrow{OA}\times\overrightarrow{OC}|=\sqrt{35}$

$h=\dfrac{10}{\sqrt{35}}=2\sqrt{\dfrac{5}{7}}$

70. (D)

Sol. Let $\vec{c}=\lambda\vec{a}+\mu\vec{b}+\nu(\vec{a}\times\vec{b})$, then

$\vec{a}\cdot\vec{c}=\lambda\Rightarrow\lambda=\cos\theta$

Similarly $\mu=\cos\theta$

also, $|\vec{c}\,|^2=\lambda^2+\mu^2+\nu^2|\vec{a}\times\vec{b}\,|^2$

$\Rightarrow 1=2\lambda^2+\nu^2\,[\,|\vec{a}\,|^2|\,\vec{b}\,|^2-(\vec{a}.\vec{b})^2\,]=2\lambda^2+\nu^2 \quad [\because\lambda=\mu]$

$\therefore\ \nu^2=1-2\lambda^2=1-2\cos^2\theta=-\cos 2\theta$

Now $\nu^2\geq 0\Rightarrow\cos 2\theta\leq\ 0\Rightarrow\dfrac{\pi}{4}\leq\theta\leq\dfrac{3\pi}{4}$

71. (D)

Sol. Let $\vec{c}=\lambda(\vec{a}+\vec{b})+\mu(\vec{a}+\vec{b})$

taking dot product with $\vec{a}$, we get

$2=\lambda\,[4+2]\Rightarrow\lambda=\dfrac{1}{3}$

squaring $4=\lambda^2\,[4+4+4]=\mu^2.\,12$

$\Rightarrow\mu^2=\dfrac{2}{9}$

$\Rightarrow$ taking dot product with $(\vec{a} \times \vec{b})$

$[(\vec{a} \times \vec{b}).\ \vec{c}\,] = \mu(12)$

$= \dfrac{\sqrt{2}}{3} \times 12 = 4\sqrt{2}$

$\therefore$ volume of tetrahedron is equal to $1/6.\ 4\sqrt{2} = \dfrac{2\sqrt{2}}{3}$

72. (D)

Sol. $\vec{x} \times \vec{a} + (\vec{x} \cdot \vec{b})\vec{c} = \vec{d}$

$\therefore \{\vec{x} \times \vec{a} + (\vec{x} \cdot \vec{b})\vec{c}\} \times \vec{c} = \vec{d} \times \vec{c}$

or $(\vec{x} \times \vec{a}) \times \vec{c} + (\vec{x} \cdot \vec{b})(\vec{c} \times \vec{c}) = \vec{d} \times \vec{c}$

$= (\vec{x} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{c})\vec{x} = (\vec{d} \times \vec{c})$

$\vec{a} \times \{(\vec{x} \cdot \vec{c})\,\vec{a} - (\vec{a} \cdot \vec{c})\vec{x}\} = \vec{a} \times (\vec{d} \times \vec{c})$

$= -(\vec{a} \cdot \vec{c})(\vec{a} \times \vec{x}) = \vec{a} \times (\vec{d} \times \vec{c}) \quad \because \vec{a} \times \vec{a} = 0$

$\vec{x} \times \vec{a} = \dfrac{\vec{a} \times (\vec{d} \times \vec{c})}{\vec{a}.\vec{c}}$

$\vec{a} \times (\vec{x} \times \vec{a}) = \vec{a} \times \dfrac{\vec{a} \times (\vec{d} \times \vec{c})}{\vec{a}.\vec{c}}$

$(\vec{a} \cdot \vec{a})\vec{x} - (\vec{a} \cdot \vec{x})\vec{a} = \vec{a} \times \dfrac{\vec{a} \times (\vec{d} \times \vec{c})}{\vec{a}.\vec{c}}$

$\vec{a}^2\,\vec{x} = (\vec{a} \cdot \vec{x})\vec{a} + \vec{a} \dfrac{\vec{a} \times (\vec{d} \times \vec{c})}{\vec{a}.\vec{c}}$

$\vec{x} = \dfrac{(\vec{a}.\vec{x})\vec{a}}{\vec{a}^2} + \vec{a} \times \dfrac{\vec{a} \times (\vec{d} \times \vec{c})}{(\vec{a}.\vec{c})\vec{a}^2}$

$\vec{x} = \lambda\,\vec{a} + \vec{a} \times \dfrac{\vec{a} \times (\vec{d} \times \vec{c})}{(\vec{a}.\vec{c})\vec{a}^2} \Rightarrow \lambda = \dfrac{\vec{a}.\vec{x}}{\vec{a}^2}$

73. (B)

Sol. $\vec{b}$ perpendicular $\vec{c} \Rightarrow \vec{b} \cdot \vec{c} = 0$

$\Rightarrow \tan^2\alpha - \tan\alpha - 6 = 0$

$\Rightarrow \tan\alpha = 3, -2$

also make an obtuse angle with z- axis therefore $\vec{a} \cdot \hat{k} < 0 \Rightarrow \sin 2\alpha < 0$ if $\tan\alpha = 3$ then $\sin 2\alpha = \dfrac{2\tan\alpha}{1+\tan^2\alpha} = \dfrac{4}{3} > 0$

Now $\tan 2\alpha > 0$, $\sin 2\alpha < 0 \Rightarrow \alpha \in$ third quadrant and $\tan\alpha = -2$

$\Rightarrow \tan(\pi - \alpha) = 2 \Rightarrow \alpha = (2n+1)\pi - \tan^{-1} 2,\ n \in I$

74. (D)

Sol. The given lines intersect, if the shortest distance between the lines is zero.

We know that the shortest distance between the lines $r = a_1 + (\lambda\,\vec{b}_1)$ and $r = a_2 + \lambda b_2$ is

$$\frac{|(a_1 - a_2).b_1 \times b_2|}{|b_1 \times b_2|}$$

So the shortest distance between the given lines is zero if

$(i - j - (2i - j)\,.\,(2i + k) \times (i + j - k) = 0$

$$\text{L.H.S.} = \begin{vmatrix} -1 & 0 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 1 \neq 0$$

Hence the given lines do not intersect.

75. (D)

Sol. The equations of straight line can be rewritten as

$x = ay + b,\ z = cy + d \Rightarrow \dfrac{x-b}{a} = \dfrac{y-0}{1} = \dfrac{z-d}{c}$ and $x = a\,y + b\,,\ z = c\,y + d$

$\Rightarrow \dfrac{x-b'}{a'} = \dfrac{y-0}{1} = \dfrac{z-d'}{c'}$

The above lines are perpendicular if $aa\ + 1.1 + cc\ = 0 \Rightarrow aa\ + cc\ + 1 = 0$.

76. (C)

Sol. Line of shortest distance will be along $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$

$\therefore x = (\hat{i} + 2\hat{j} + 2\hat{k})\,.\dfrac{(-\hat{i} + 2\hat{j} - \hat{k})}{\sqrt{6}} = \dfrac{1}{\sqrt{6}}$

$\Rightarrow \cos^{-1}(\cos\sqrt{6}\,x) = \cos^{-1}(\cos 1) = 1$

77. (C)

Sol. $\vec{a} + \vec{b} + \vec{c} + \vec{d} = (\alpha+1)\,\vec{d} = (\beta+1)\,\vec{a} \quad \Rightarrow \vec{d} = \dfrac{\beta+1}{\alpha+1}\,\vec{a}$

So $\vec{a} + \vec{b} + \vec{c} = \alpha\,\vec{d} = \alpha\left(\dfrac{\beta+1}{\alpha+1}\right)\vec{a}$

$\Rightarrow \vec{a}\left\{1 - \dfrac{\beta+1}{\alpha+1}\alpha\right\} + \vec{b} + \vec{c} = 0$

$\Rightarrow [\vec{a}\ \vec{b}\ \vec{c}\,] \neq 0 \Rightarrow \alpha = -1$

$\Rightarrow \vec{a} + \vec{b} + \vec{c} + \vec{d} = 0$

78. (D)

Sol. Dr's of AB 1, 4, 2 and of CD 2, 1, 9

angle between AB and CD

$= \cos^{-1}\left(\dfrac{1.2 + 4.1 + 2.9}{\sqrt{1^2 + 4^2 + 2^2}\,\sqrt{2^2 + 1^2 + 9^2}}\right)$

$= \cos^{-1}\left(\dfrac{24}{\sqrt{1806}}\right)$

equation of plane $a(x-2) + b(y-3) + c(z-4) = 0$

Hence (A) is not true.

It passes through B and C Þ $a = 2k,\ b = -5k,\ c = 9k$

Þ $2x - 5y + 9z - 25 = 0$

\ dr's of plane ABC is 2, –5, 9

dr's of line AB is 4, 0, 8

angle between AD and plane ABC $= \sin^{-1}\sqrt{\dfrac{8}{11}}$

Hence (B) is not true.

dr's of BD is 3, –4, 6 so BD $\dfrac{x-1}{3} = \dfrac{y+1}{-4} = \dfrac{z-1}{6}$

Hence (C) is not true.

perpendicular distance from D to ABC

$= \left|\dfrac{2(-2) - 5(3) + 9(-4) - 25}{\sqrt{2^2 + (-5)^2 + 9^2}}\right|$

$= \dfrac{80}{\sqrt{110}}$

79. (C)

Sol. For vectors $\vec{a}$ and $\vec{b}$ to be inclined at an obtuse angle

$\vec{a}\cdot\vec{b}<0,\ x\in(0,\infty)$

$c(\log_2 x)^2-12+6c\log_2 x<0\ \forall\ x\in(0,\infty)$

$cy^2+6<cy-12<0 \qquad y\in R\ y=\log_2 x$

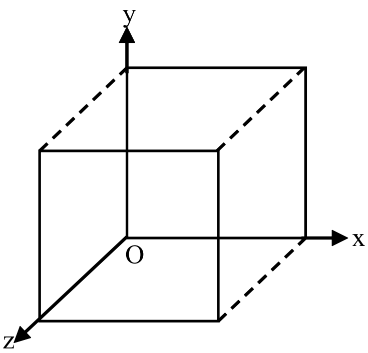
$c<0$ and $36c^2+48c<0$

$c<0 \qquad c(3c+4)<0$

$c\in(-4/3,0)$

80. (B)

Sol. dr's of diagonal through the origin are (1, 1, 1)



dr's of edge, parallel to x-axis and in the plane $z=0$, are (1, 0, 0)

Hence d.c. of the line of shortest distance are $\left(0,\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$

Projection of line segment joining (1, 1, 1) and (1, 1, 0) on the line of shortest distance

$$=\sqrt{\left(1-\frac{1}{2}\right)^2+\left(0-\frac{1}{2}\right)^2+\left(\frac{1}{2}-\frac{1}{2}\right)^2}=\frac{1}{\sqrt{2}}$$

81. 4

Sol. $\because\ \overline{p}\cdot\overline{z}=a_1[\overline{x}\ \overline{y}\ \overline{z}] \qquad \overline{p}\cdot\overline{x}=a_2[\overline{x}\ \overline{y}\ \overline{z}]$

$\therefore\ 4(a_1+a_2)=\overline{p}\cdot(\overline{z}+\overline{x})=(a_1+a_2)[\overline{x}\ \overline{y}\ \overline{z}]$

$\Rightarrow[\overline{x}\ \overline{y}\ \overline{z}]=4$

82. 8

Sol. $\vec{a}\cdot\vec{b}=4+4+\lambda=0\Rightarrow\lambda=-8$

83. 3

Sol. $\overrightarrow{AB}=\vec{B}-\vec{A}=\hat{i}-2\hat{j}+2\hat{k},\ |\overrightarrow{AB}|=\sqrt{1+4+4}=3$

84. 600

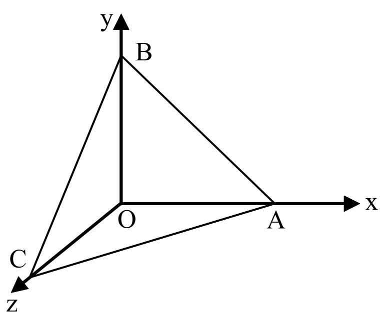
Sol. Given plane meets axis at the points

A(20, 0, 0), B(0, 15, 0), C (0, 0, –12)

$\overrightarrow{OA}=20\hat{i},\ \overrightarrow{OB}=15\hat{j},\ \overrightarrow{OC}=-12\hat{k}$

$\text{Volume}=\frac{1}{6}\left|[\overrightarrow{OA}\ \overrightarrow{OB}\ \overrightarrow{OC}]\right|$

$=\frac{1}{6}\times20\times15\times12=600$



85. 1

Sol. Let the vertices A, B, C, D quadrilateral be

$(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3)$ & (x_4, y_4, z_4) and the equation of plane PQRS be

$u \equiv ax + by + cz + d = 0$

let $u_r = a_r x + b_r y + c_r z + d$ where r = 1, 2, 3, 4

Then $\frac{AP}{PB} \cdot \frac{BQ}{QC} \cdot \frac{CR}{RD} \cdot \frac{DS}{SA}$

$= \left(-\frac{u_1}{u_2}\right)\left(-\frac{u_2}{u_3}\right)\left(-\frac{u_3}{u_4}\right)\left(-\frac{u_4}{u_1}\right) = 1.$

86. 24

Sol. Any point on the ellipse $\frac{x^2}{(3\sqrt{2})^2} + \frac{y^2}{(4\sqrt{2})^2} = 1$ can be taken as $(3\sqrt{2}\cos\theta, 4\sqrt{2}\sin\theta)$ and the slope of the tangent

$= -\frac{b^2 x}{a^2 y}$

$= -\frac{32(3\sqrt{2}\cos\theta)}{18(4\sqrt{2}\sin\theta)} \qquad = -\frac{4}{3}\cot\theta \qquad .... (1)$

Given slope of the tangent $= -\frac{4}{3}$ .... (2)

From equations (1) and (2)

$\cot\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$

Hence the equation of the tangent is $\frac{x.\frac{1}{\sqrt{2}}}{3\sqrt{2}} + \frac{y.\frac{1}{\sqrt{2}}}{4\sqrt{2}} = 1$

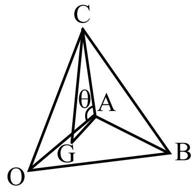
(i.e.) $\frac{x}{6} + \frac{y}{8} = 1$

Hence A = (6, 0), B = (0, 8)

Area of $\Delta OAB = \frac{1}{2} \times 6 \times 8 = 24$ sq. units

87. 5

Sol.



Let OABC be the tetrahedron. Let G be the centroid of the face OAB, then $GA = \frac{1}{\sqrt{3}} AC$.

Then $\cos\theta = \frac{GA}{CA} = \frac{1}{\sqrt{3}}$

$\therefore \cos^2\theta = \frac{1}{3} \qquad \therefore \qquad a = 1$ and $b = 3$

$\therefore \qquad 10a + b = 13$

88. 3

Sol. Area of quadrilateral OABC = $\Delta OAC + \Delta ABC$

$= \frac{1}{2}\left|\overline{OA} \times \overline{AC}\right| + \frac{1}{2}\left|\overline{AB} \times \overline{BC}\right|$

$= \frac{1}{2}|\vec{a} \times (\vec{b} - \vec{a})| + \frac{1}{2}|(2\vec{a} + 10\vec{b} - \vec{a})$

$\times (\vec{b} - 2\vec{a} - 10\vec{b})|$

$= \frac{1}{2}|\vec{a} \times \vec{b}| + \frac{1}{2}|(\vec{a} + 10\vec{b}) \times (2\vec{a} + 9\vec{b})|$

$= \frac{1}{2}|\vec{a} \times \vec{b}| + \frac{11}{2}|\vec{a} \times \vec{b}| = 6|\vec{a} \times \vec{b}|$

$|\vec{a} \times \vec{b}| = m$

$\ell = 2\lambda m$

$\Rightarrow 6|\vec{a} \times \vec{b}| = 2\lambda|\vec{a} \times \vec{b}|$

$\Rightarrow \lambda = 3.$

89 1

Sol. $\hat{i} \times \left[(\vec{a} - \hat{j}) \times \hat{i}\right] = (\hat{i}.\hat{i})(\vec{a} - \hat{j}) - (\hat{i}.(\vec{a} - \hat{j}))\hat{i}$

$= \vec{a} - \hat{j} - (\hat{i}.\vec{a})\hat{i}$

$\therefore \vec{a} - \hat{j} - (\hat{i}.\vec{a})\hat{i} + \vec{a} - \hat{k} + (\hat{j}.\vec{a})\hat{j} + \vec{a} - \hat{i} - (\hat{k}.\vec{a})\hat{k} = 0$

$3\vec{a} - (\hat{i} + \hat{j} + \hat{k}) - \vec{a} = 0$

$\vec{a} = \frac{1}{2}(\hat{i} + \hat{j} + \hat{k}) = x\hat{i} + y\hat{j} + z\hat{k}$

$\therefore x = y = z = \frac{1}{2}$

$\Rightarrow 8(x^3 - xy + zx) = 8(x^3 - x^2 + x^2) = 8 \times \frac{1}{8} = 1$

90. 1

Sol. Given planes are

$x - cy - bz = 0$ ... (i)

$cx - y + az = 0$ ... (ii)

$bx + ay - z = 0$ ... (iii)

equation of plane passing through the line of intersection of plane (i) and (ii) may be taken as

$(x - cy - bz) + \lambda(cx - y + az) = 0$

$\Rightarrow (1 + \lambda c)x - y(c + \lambda) + z(a\lambda - b) = 0$ ....(iv)

If plane (iii) and (iv) are same then

$$\frac{1 + \lambda c}{b} = \frac{-(c + \lambda)}{a} = \frac{a\lambda - b}{-1}$$

$\Rightarrow \lambda = -\frac{a + bc}{ac + b} = -\frac{ab + c}{1 - a^2}$

$\Rightarrow a - a^3 + bc - a^2bc = a^2bc + ac^2 + ab^2 + bc$

$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$