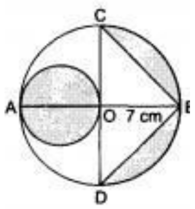
CBSE Test Paper 05

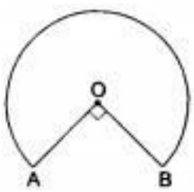
Chapter 12 Area Related to Circle

1. The diameter of the circle whose area is 301.84 cm^2 is **(1)**
   a. 14.2 cm
   b. 12.8 cm
   c. 19.6 cm
   d. 15.6 cm
2. A piece of wire 20cm long is bent into the form of an arc of a circle subtending an angle of 60 at its centre. The radius of the circle is **(1)**
   a. $\frac{20}{6+\pi}$ cm
   b. $\frac{30}{6+\pi}$ cm
   c. $\frac{60}{6+\pi}$ cm
   d. $\frac{15}{6+\pi}$ cm
3. The radius of a circle is 20 cm Three more concentric circles are drawn inside it in such a way that it is divided into four parts of equal area. The radius of the largest of the three concentric circles is **(1)**
   a. $14\sqrt{3}cm$
   b. $10\sqrt{3}\ cm$
   c. $8\sqrt{3}cm$
   d. None of these.
4. If the area of a circle is equal to the area of a square, then the ratio of their perimeters is **(1)**
   a. $2\ :\ \pi$
   b. 1 : 2
   c. $\sqrt{\pi}\ :\ 2$
   d. $\pi : 2$
5. A bicycle wheel makes 5000 revolutions in moving 11km. The diameter of the wheel is **(1)**
   a. 100cm
   b. 35cm
   c. 140cm
   d. 20cm
6. Find the area of a sector of angle p (in degrees) of a circle with radius R. **(1)**

7. A rope by which a cow is tethered is increased from 16m to 23m. How much additional ground does it have now to graze? **(1)**
8. To warn ships for underwater rocks, a lighthouse spreads a red-coloured light over a sector of angle 72° to a distance of 15 km. Find the area of the sea over which the ships are warned. [Use π = 3.14.] **(1)**
9. Find the area of a quadrant of a circle whose circumference is 88 cm. **(1)**
10. If the perimeter of a protractor is 72 cm, then calculate its area. **(1)**
11. In the given figure, AB and CD are the diameters of a circle with centre O, perpendicular to each other. OA is the diameter of the smaller circle. If OB = 7 cm, find the area of the shaded region. **(2)**



12. In the given figure, the shape of the top of a table is that of a sector of a circle with centre O and $\angle AOB$ = 90°. If AO = OB = 42 cm then find the perimeter of the top of the table. **(2)**



13. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find **(3)**
    i. the area of that part of the field in which the horse can graze.
    ii. the increase in the grazing area if the rope were 10 m long instead of 5 m (Use π = 3.14)

14. Find the area enclosed between two concentric circles of radii 3.5 cm and 7 cm. A third concentric circle is drawn outside the 7 cm circle, such that the area enclosed between it and the 7 cm circle is same as that between the two inner circles. Find the radius of the third circle correct to one decimal place. **(3)**

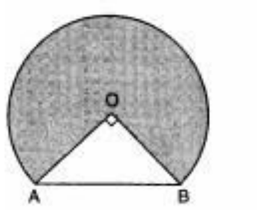
15. The cost of fencing a circular field at the rate Rs 24 per metre is Rs 5280. The field is to be ploughed at the rate of Rs 0.50 per m^2. Find the cost of ploughing the field. **(3)**

16. Below figure shows the cross-section of railway tunnel. The radius OA of the circular part is 2 m. If $\angle$AOB = 90°, calculate

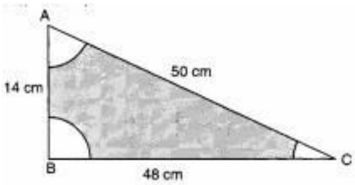
i. the height of the tunnel

ii. the perimeter of the cross-section

iii. the area of the cross-section **(3)**



17. With vertices A, B and C of a triangle ABC as centres, arcs are drawn with radii 5 cm each as shown in Fig. If AB = 14 cm, BC = 48 cm and CA = 50 cm, then find the area of the shaded region. (Use π = 3.14). **(3)**



18. A path of 4 m width runs round a semi-circular grassy plot whose circumference is $163\frac{3}{2}$ m Find:

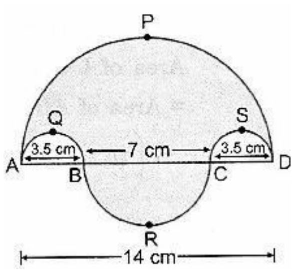
i. the area of the path

ii. the cost of gravelling the path at the rate of Rs 1.50 per square metre

iii. the cost of turfing the plot at the rate of 45 paise per m^2. **(4)**

19. A chord of a circle subtends an angle of θ at the centre of the circle. The area of the minor segment cut off by the chord is one eighth of the area of the circle. Prove that $8\sin\frac{\theta}{2}\cos\frac{\theta}{2}+\pi=\frac{\pi\theta}{45}$. **(4)**

20. Find the area of the shaded region in Figure, $\widehat{APD}$, $\widehat{AQB}$, $\widehat{BRC}$ and $\widehat{CSD}$, are semi-circles of diameter 14 cm, 3.5 cm, 7 cm and 3.5 cm respectively.$\left(\text{Use } \pi=\frac{22}{7}\right)$. **(4)**



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Solution

1. c. 19.6 cm

Explanation: Let diameter of the circle be d cm. $= 384.16$

$\therefore$ Area = $\pi r^2 = \pi\left(\frac{d}{2}\right)^2$

2. c.

$\Rightarrow 301.84 = \frac{22}{7} \times \frac{d^2}{4}$

$\Rightarrow d^2 \quad \frac{301.84 \times 4 \times 7}{22}$

$\Rightarrow d^2$

$\Rightarrow d = 19.6$ cm

$\frac{60}{6+\pi}$ cm

Explanation: Given: Length of arc + 2 $\times$ Radius = 20 cm $\Rightarrow$

$\frac{\theta}{360^\circ} \times 2\pi r + 2r = 20$

$\Rightarrow \frac{60^\circ}{360^\circ} \times 2\pi r + 2r = 20$

$\Rightarrow \frac{\pi r}{3} + 2r = 20$

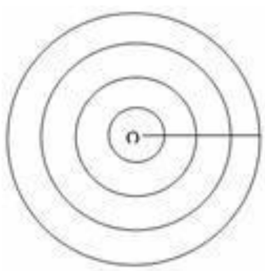
3. b.

$\Rightarrow r\left(\frac{\pi}{3} + 2\right) = 20$

$\Rightarrow r\left(\frac{6+\pi}{3}\right) = 20$

$\Rightarrow r = \frac{60}{6+\pi}$ cm

$10\sqrt{3}\ cm$

Explanation: 

Here, Radius of bigger circle (OA) = 20 cm

Let the radii of three concentric circle inscribe in the bigger circle be r, r_1 and r_2 in decreasing order of length of radius respectively.

Then according to the question,

$\pi(20)^2 - \pi r^2 = \pi r^2 - \pi r_1^2$

$\Rightarrow 400 - r^2 = r^2 - r_1^2$

$\Rightarrow 2r^2 = r_1^2 + 400$ ....(i)

Also $\pi r^2 - \pi r_1^2 = \pi r_1^2 - \pi r_2^2$

$\Rightarrow r^2 - r_1^2 = r_1^2 - r_2^2$

$\Rightarrow r_2^2 = 2r_1^2 - r^2$

...........(ii)

And $\pi r_1^2 - \pi r_2^2 = \pi r_2^2 \Rightarrow r_1^2 - r_2^2 = r_2^2$

$\Rightarrow r_2^2 = \frac{1}{2} r_1^2$ ....(iii)

Putting the value of inr_2^2 eq. (ii), we get

$\frac{1}{2} r_1^2 = 2r_1^2 - r^2$

$\Rightarrow r_1^2 = \frac{2}{3} r^2$

Now, putting the value ofr_1^2 in eq. (i), we get

, $2r^2 = \frac{2}{3} r^2 + 400$

$\Rightarrow 2r^2 - \frac{2}{3} r^2 = 400$

$r^2 = 100 \times 3$

$\Rightarrow r = 10\sqrt{3}$ cm or 17.32 m (approx)

4. c. $\sqrt{\pi} : 2$

Explanation: Let the radius of the circle be r and the side of the square be a.

Then according to the question, $\pi r^2 = \Rightarrow a a^2 = r\sqrt{\pi}$ ....(i)

Now, Ratio of their perimeters = $\frac{2\pi r}{4a}$

$\Rightarrow$ Ratio of their perimeters = $\frac{\pi r}{2a} = \frac{\pi r}{2r\sqrt{\pi}} = \frac{\sqrt{\pi}}{2}$

Ratio of their perimeters = $\sqrt{\pi} : 2$

5. d. 20cm

Explanation: Let diameter of the wheel be d cm.

Given: Distance = 11 km = 1100000 cm

No. of Revolutions = $\frac{\text{Total distance}}{\text{Circumference of wheel}}$

$\Rightarrow 5000 = \frac{1100000 \times 7}{22 \times d}$

$\Rightarrow d = \frac{1100000 \times 2}{5000 \times 22}$

$\Rightarrow$ d= 100000/5000

= 20 cm

6. Area of sector = $\frac{\theta}{360} \times \pi r^2$

where θ= angle, r = radius of circle

Here, we have

θ= p and radius = R

Putting these in formula,

Area of sector = $= \frac{p}{360} \times \pi R^2$

7. Area grazed by the cow = $\pi(16)^2$

If the length of the rope is increased, area grazed by cow = $\pi \times (23)^2$

Hence, additional area grazed by cow

= $\pi \times (23)^2$ - $\pi(16)^2$

= 858 m^2

8. Required area = area of the sector in which, r = 15 km

and θ = 72°

$$\equiv \left(\frac{\pi r^2 \theta}{360}\right)$$
$$= \left(3.14 \times 15 \times 15 \times \frac{72}{360}\right) \mathrm{km}^2$$
$$= \left(\frac{314 \times 45}{100}\right) \mathrm{km}^2$$
$$= \frac{1413}{10} \mathrm{km}^2$$

=141.3 km^2.

9. We know that circumference of the circle $= 2\pi r$.

$$\Rightarrow 88 = 2 \times \frac{22}{7} \times r$$
$$\Rightarrow 88 \times \frac{7}{44} = r$$
$$\Rightarrow 2 \times 7 = r \Rightarrow r = 14\ cm$$

Now, Area of circle = πr^2 = $\frac{22}{7} \times 14 \times 14$ = 616 cm^2

10. Given, Perimeter of semi-circular protactor = 72

Perimeter of a semi-circular protactor = Perimeter of semi-circle = $2r + \pi r = 72$

$$\Rightarrow r = \frac{72}{\pi + 2} = \frac{72 \times 7}{36}$$

$$\Rightarrow r = 14 cm$$

Area of protactor = $\frac{1}{2} \pi r^2 = \frac{1}{2} \times \frac{22}{7} \times (14)^2 = 308$

∴Area of semi-circular protactor = 308 cm^2

11. Clearly, the diameter of the larger circle is 14 cm and the diameter of the smaller circle is 7 cm.

So, the radius of the larger circle is 7 cm and that of the smaller circle is 3.5 cm.

Area of the shaded region,

= {(area of smaller circle) + (area of larger semicircle)} - (area of $\triangle CBD$)

$= \left[\left\{\pi \times \left(\frac{7}{2}\right)^2\right\} + \left\{\frac{1}{2} \times \pi \times 7 \times 7\right\} - \left\{\frac{1}{2} \times CD \times OB\right\}\right] \mathrm{cm}^2$

$= \left\{\left(\frac{22}{7} \times \frac{49}{4}\right) + \left(\frac{1}{2} \times \frac{22}{7} \times 49\right) - \left(\frac{1}{2} \times 14 \times 7\right)\right\} \mathrm{cm}^2$

$= \left(\frac{77}{2} + 77 - 49\right) \mathrm{cm}^2$

= (38.5 + 28)cm^2

= 66.5 cm^2

12. $90^{\circ}\angle AOB =$

$AO = OB = 42cm$

$\Rightarrow$ Radius of a circle $= 42cm$

∴ Required perimeter

= Circumference of a circle- [Length of arc AB + (AO + OB)]

$= \left\{\left(2 \times \frac{22}{7} \times 42\right) - \left(2 \times \frac{22}{7} \times 42 \times \frac{90}{360}\right) + (42 + 42)\right\} \mathrm{cm}$

$= \left\{\left(2 \times \frac{22}{7} \times 42\right) - \left(2 \times \frac{22}{7} \times 42 \times \frac{90}{360}\right) + (42 + 42)\right\} \mathrm{cm}$

$= (264 - 66 + 84)cm$

$= 282cm$

13. i. The area of that part of the field in which the horse can graze if the length of the rope is 5cm

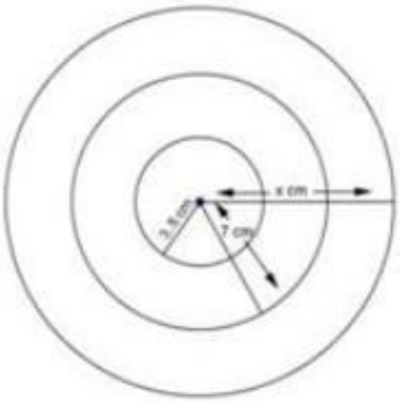
$= \frac{1}{4}\pi r^2 = \frac{1}{4} \times 3.14 \times (5)^2 = \frac{1}{4} \times 78.5 = 19.625 \mathrm{m}^2$

ii. The area of that part of the field in which the horse can graze if the length of the rope is 10 m

$= \frac{1}{4}\pi r^2 = \frac{1}{4} \times 3.14 \times (10)^2 = 78.5 \mathrm{m}^2$

∴ The increase in the grazing area

= 78.5 - 19.625 = 58.875cm^2

14. 

Radius of first circle = 3.5 cm

Radius of second circle = 7 cm

Let radius of third circle = x cm

Area between first and second circle

$= \pi(7)^2 - \pi(3.5)^2$

$= 36.75\pi cm^2$

Area between second and third circle

$= \pi(x)^2 - \pi(7)^2$

$= \pi x^2 - 49\pi$

According to question

Area between first and second circle = Area between second and third circle

$36.75\pi = \pi x^2 - 49\pi$

$36.75\pi + 49\pi = \pi x^2$

$85.75\pi = \pi x^2$

$\Rightarrow x^2 = 85.75$

$x = \sqrt{85.75} = 9.26cm$

15. We have, Rate of fencing = Rs 24 per metre and, Total cost of fencing = Rs 5280

$\therefore$ Length of the fence = $\frac{\text{Total cost}}{\text{Rate}} = \frac{5280}{24}$ = 220 metre

$\Rightarrow$ Circumference of the field = 220 metre

$\Rightarrow 2\pi r$ = 220, where r is the radius of the field

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 220$

$\Rightarrow \quad r = \frac{220 \times 7}{22 \times 2} = 35$

Area of the field = $\pi r^2 = \frac{22}{7} \times 35 \times 35 \mathrm{m}^2 = 22 \times 5 \times 35 \mathrm{m}^2$

It is given that the field is ploughed at the rate of Rs 0.50 per m^2

Cost of ploughing the field = Rs $(22 \times 5 \times 35 \times 0.50)$= Rs 1925

16. Radius OA = OB = 2m

$\angle AOB$ = 90°

In $\angle AOB$, by pythagoras theorem

$AB^2 = OA^2 + OB^2$

$\Rightarrow AB^2 = 2^2 + 2^2$

$\Rightarrow AB^2 = 8$

$\Rightarrow AB = \sqrt{8} = 2\sqrt{2}$ m

Area of $\Delta AOB = \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 2 \times 2 = 2\text{m}^2$

Again area of $\Delta AOB = \frac{1}{2} \times AB \times OC = \frac{1}{2} \times 2\sqrt{2} \times OC = \sqrt{2} \times OC\text{m}^2$

$\therefore \sqrt{2} \times \text{OC} = 2$

$\Rightarrow OC = \frac{2}{\sqrt{2}} = \sqrt{2}\text{m}$

i. $\therefore$ height of tunnel = DO + OC

= (2 + $\sqrt{2}$)m

ii. perimeter of cross- section = AB + area of major arc AB

$= 2\sqrt{2} + \frac{270}{360} \times 2\pi r$

$= 2\sqrt{2} + \frac{3}{4} \times 2\pi \times 2$

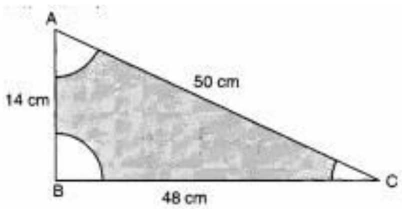
$= (2\sqrt{2} + 3\pi)m$

iii. the area of corss - section = Area of major sector + Area of $\triangle AOB$

$= \frac{270^\circ}{360} \times \pi(2)^2 + 2$

$= \frac{3}{4}\pi \times 4 + 2$

$= (3\pi + 2)m^2$

17. 

In $\triangle$ABC, we have,

a = BC = 48 cm, b = CA = 50 cm and c = AB = 14 cm

Let s be the semi-perimeter of $\triangle$ABC. Then,

$s = \frac{a+b+c}{2} = \frac{48+50+14}{2} = 56\text{cm}$

Let $\triangle$ be the area of $\triangle$ABC. Then, by Heron's formula

$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{56 \times 8 \times 6 \times 42}\text{cm}^2 = 336\text{cm}^2$

Let A_1, A_2 and A_3 be the areas of sectors with sector angles A, B and C respectively and sector radius r = 5 cm. Then,

$$A_1 = \frac{A}{360} \times \pi r^2 = \frac{A}{360} \times \pi \times 5^2 \text{cm}^2 = \frac{A}{360} \times 25\pi \text{cm}^2$$
$$A_2 = \frac{B}{360} \times \pi r^2 = \frac{B}{360} \times \pi \times 5^2 \text{cm}^2 = \frac{B}{360} \times 25\pi \text{cm}^2$$
$$A_3 = \frac{C}{360} \times \pi r^2 = \frac{C}{360} \times \pi \times 5^2 \text{cm}^2 = \frac{C}{360} \times 25\pi \text{cm}^2$$

$\therefore$ $A_1 + A_2 + A_3$ = $\left(\frac{A}{360} \times 25\pi + \frac{B}{360} \times 25\pi + \frac{C}{360} \times 25\pi\right) \text{cm}^2$

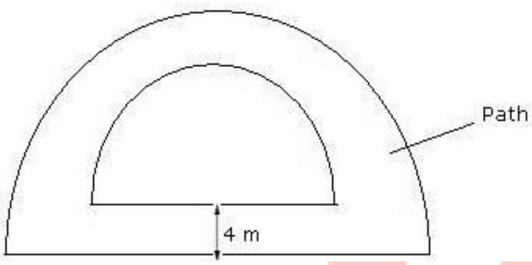
$= (A + B + C) \times \frac{25\pi}{360} \text{cm}^2$

$= \frac{180}{360} \times 25\pi \text{cm}^2 = \frac{25\pi}{2} \text{cm}^2 = \frac{25 \times 3.14}{2} \text{cm}^2$ = 39.25 cm^2

Let A be the area of the shaded region. Then,

A = Area of $\triangle$ABC - ($A_1 + A_2 + A_3$) = (336 - 39.25) cm^2 = 296.75 cm^2

18.



Let x be the radius of the semi-circular grassy plot.

Given,

Circumference of grassy plot = $163\frac{3}{7} m$

$\Rightarrow 2r + \pi r = 163\frac{3}{7} m = \frac{1144}{7} m$

$\Rightarrow (2 + \pi) r = \frac{1144}{7} m$

$\Rightarrow \frac{36}{7} r = \frac{1144}{7} m$

$\Rightarrow r = \frac{1144}{36} = \frac{286}{9} m$

$\therefore$ Radius of semi-circular grassy plot $= \frac{286}{9} m$

Then, radius of outer semi-circle $= \frac{286}{9} m + 4m = \frac{322}{9} m$

i. Area of path = Area of outer semi-circle - Area of inner semi-circle

$= \frac{1}{2}\pi\left(\frac{322}{9}\right)^2 - \frac{1}{2}\pi\left(\frac{286}{6}\right)^2$

$= \frac{1}{2}\pi\left(\frac{322}{9} + \frac{286}{9}\right)\left(\frac{322}{9} - \frac{286}{9}\right)$

$= \frac{1}{2} \times \frac{22}{7} \times \frac{608}{9} \times \frac{36}{9}$

= 424.63m^2

ii. Rate of gravelling the path = Rs.1.50 per m^2

$\therefore$ Total cost = Rs. 1.50 $\times$ 424.63

= Rs. 636.95

iii. Area of plot $= \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times \frac{286}{9} \times \frac{286}{9} m^2$

= 1586.87m^2

Rate of turfing the plot = 45 paise per m^2

$\therefore$ Total cost = 1586.87 $\times$ 45paise

= 71409.15 paise

= Rs. 714.09

19. Given

Area of minor segment cut off by $AB = \frac{1}{8} \times$ Area of circle

$\Rightarrow \quad \frac{\theta}{360^\circ} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta = \frac{1}{8} \times \pi r^2$

$\Rightarrow \quad \frac{\theta}{360^\circ} \times \pi r^2 = \frac{1}{8}\pi r^2 + \frac{1}{2} r^2 \sin\theta$

$\Rightarrow \quad \frac{\theta}{360^\circ} \times \pi = \frac{\pi}{8} + \frac{1}{2}\sin\theta \quad \left[\text{Divide by } r^2\right]$

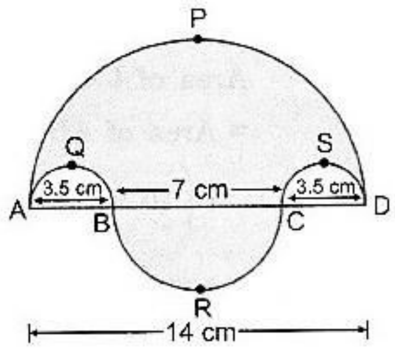
$\Rightarrow \quad \frac{\pi\theta}{45^\circ} = \pi + 4\sin\theta \quad$ [Multiply by 8]

$\Rightarrow \quad \frac{\pi\theta}{45^\circ} = \pi + 4 \times 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \quad [\sin 2\theta = 2\sin\theta\cos\theta]$

$\Rightarrow \quad \frac{\pi\theta}{45^\circ} = \pi + 8\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

Hence Proved

20.



Diameter of the largest semi circle = 14 cm (3.4 cm + 7 cm + 3.5 cm)

Therefore, the radius will be $\frac{14}{2}$ = 7 cm

Diameter of two equal un-shaded semicircle = 3.5 cm (given)

Therefore, Radius of each circle = $\frac{3\cdot 5}{2}$cm

The diameter of smaller shaded semicircle = 7 cm

Therefore the radius will be $= \frac{7}{2}$cm = 3.5 cm

Area of shaded portion = area of largest semicircle + area of smaller shaded semicircle

- area of two un-shaded semicircles

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 + \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} - 2 \times \frac{22}{7} \times \frac{1}{2} \times \frac{3\cdot5}{2} \times \frac{3\cdot5}{2}$

$= \frac{1}{2} \times \frac{22}{7} \times 49 + \frac{1}{2} \times \frac{22}{7} \times \frac{49}{4} - \frac{22}{7} \times \frac{1}{2} \times 3.5 \times \frac{3\cdot5}{2}$

$= \frac{1}{2} \times \frac{22}{7} \left[49 + \frac{49}{4} - \frac{49}{8}\right]$

$= \frac{1}{2} \times \frac{22}{7} \times 49 \left[1 + \frac{1}{4} - \frac{1}{8}\right]$

$= \frac{1}{2} \times \frac{22}{7} \times 49 \left[\frac{8+2-1}{8}\right]$

$= \frac{1}{2} \times \frac{22}{7} \times 49 \left[\frac{9}{8}\right]$

$= \frac{1}{2} \times \frac{22}{7} \times \left[\frac{441}{8}\right]$

$= \frac{11}{7} \times \left[\frac{441}{8}\right]$

$= \frac{693}{8}$ sq. cm

$= 86.625 \text{ cm}^2$

Hence, the area of shaded portion is 86.625 cm^2.