CBSE Test Paper 02
Chapter 8 Introduction to Trigonometry

1. The value of $\frac{\tan 30^\circ}{\cot 60^\circ}$ is **(1)**
   a. 1
   b. $\frac{1}{\sqrt{3}}$
   c. $\frac{1}{\sqrt{2}}$
   d. $\sqrt{2}$
2. $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} =$ **(1)**
   a. $sin45^\circ$
   b. 0
   c. $cos45^\circ$
   d. $tan45^\circ$
3. The value of $tan15^\circ tan20^\circ tan70^\circ tan75^\circ$ is **(1)**
   a. 2
   b. 0
   c. 1
   d. -1
4. If x cos A = 1 and tan A = y, then the value of $x^2 - y^2$ is **(1)**
   a. -1
   b. 0
   c. 1
   d. 2
5. If $sin\ \theta = \frac{1}{2}$ and $cos\phi = \frac{1}{2}$, then the value of $(\theta + \phi)$ is **(1)**
   a. 0°
   b. 30°
   c. 90°
   d. 60°

6. In a rectangle ABCD, AB = 20cm, $\angle BAC = 60^\circ$, calculate side BC and diagonals AC and BD. **(1)**

7. Convert the given trigonometric equation in the simplest form.

$\sin 60^{\circ} \cos 30^{\circ} + \cos 60^{\circ} \sin 30^{\circ}$ **(1)**

8. Without using trigonometric tables, prove that: $(\sin 65^{o} + \cos 25^{o})(\sin 65^{o} - \cos 25^{o}) = 0$. **(1)**

9. Evaluate $(\text{cosec}^2 45^{\circ} \sec^2 30^{\circ})(\sin^2 30^{\circ} + 4\cot^2 45^{\circ} - \sec^2 60^{\circ})$ **(1)**

10. If $\cos\theta = \frac{7}{25}$, write the value of $(\tan\theta + \cot\theta)$. **(1)**

11. If $\theta = 30^{\circ}$, verify that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **(2)**

12. If A = 60° and B = 30°, verify that cos (A - B) = cos A cos B + sin A sin B. **(2)**

13. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$ **(2)**

14. If cosec A = 2, find the value of $\frac{1}{\tan A} + \frac{\sin A}{1+\cos A}$ **(3)**

15. If $a\cos\theta - b\sin\theta = x$ and $a\sin\theta + b\cos\theta = y$, prove that $a^2 + b^2 = x^2 + y^2$. **(3)**

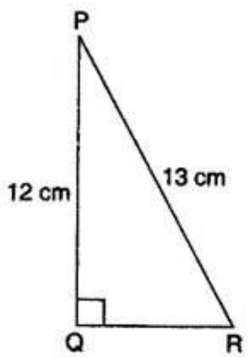
16. Find the value of x if $4\left(\frac{\sec^2 59^{\circ} - \cot^2 31^{\circ}}{3}\right) - \frac{2}{3}\sin 90^{\circ} + 3\tan^2 56^{\circ} \times \tan^2 34^{\circ} = \frac{x}{3}$ **(3)**

17. If A, B, C, are the interior angles of a ΔABC, show that $\sin\frac{B+C}{2} = \cos\frac{A}{2}$. **(3)**

18. If $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$, then find the value of $\cot^2\theta + \tan^2\theta$. **(4)**

19. Find the value of other trigonometric ratios, given that $\tan\theta = \frac{2mn}{m^2-n^2}$ **(4)**

20. In figure, find tan P - cot R. **(4)**



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Solution

1. a. 1

Explanation: Given: $\frac{\tan 30^\circ}{\cot 60^\circ}$

= $\frac{\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}}}$ = 1

2. b. 0

Explanation: Given: $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$

= $\frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1}$ = 0/2

= 0

3. c. 1

Explanation: Given: tan 15^o tan 20^o tan 70^o tan 75^o

= tan 15^o tan 20^o tan (90^o - 20^o) tan (90^o - 15^o)

= tan 15^o tan 20^o cot 20^o cot 15^o

= (tan 15^o cot 15^o) (tan 20^o cot 20^o)

= 1 $\times$ 1 = 1

4. c. 1

Explanation: Given: $x \cos \mathrm{A} = 1 \Rightarrow x = \frac{1}{\cos \mathrm{A}} = \sec \mathrm{A}$

And $\tan \mathrm{A} = y$

$\therefore x^2 - y^2 = \sec^2 \mathrm{A} - \tan^2 \mathrm{A} = 1$

$[\because \sec^2\theta - \tan^2\theta = 1]$

5. c. 90°

Explanation: Given: $\sin\theta = \frac{1}{2}$

$\Rightarrow \sin\theta = \sin 30^\circ$

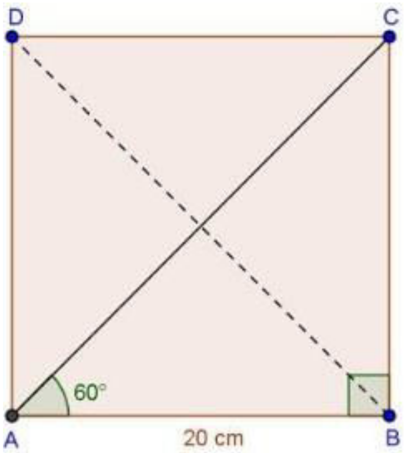
$\Rightarrow \theta = 30^\circ$

And $\cos\phi = \frac{1}{2}$

$\Rightarrow \cos\phi = \cos 60^\circ$

$\Rightarrow \theta = 60^\circ$

$$\theta + \phi = 30^\circ + 60^\circ = 90^\circ$$

6. 

Since ABCD is a rectangle

then $\angle ABC = 90^\circ$

Now, in ΔABC

$\tan(\angle BAC) = \frac{BC}{AB}$ and $\cos(\angle BAC) = \frac{AB}{AC}$

$\Rightarrow \tan 60^\circ = \frac{BC}{20}$ and $\cos 60^\circ = \frac{20}{AC}$

$\Rightarrow \sqrt{3} = \frac{BC}{20}$ and $\frac{1}{2} = \frac{20}{AC}$

$\Rightarrow BC = 20\sqrt{3}cm$ and AC = 40 cm

We know that in a rectangle, diagonals are equal

∴ BD = AC = 40 cm

7. We have,

$\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1$

8. $(\sin 65^\circ + \cos 25^\circ)(\sin 65^\circ - \cos 25^\circ) = 0$

LHS = $(\sin 65^\circ + \cos 25^\circ)(\sin 65^\circ - \cos 25^\circ)$

$= \sin^2(65^\circ) - \cos^2(25^\circ)$

$= \sin^2(90^\circ - 25^\circ) - \cos^2(25^\circ)$

$= \cos^2(25^\circ) - \cos^2(25^\circ)$

= 0 = RHS

9. We know that, cosec45°=√2, sec30°=(2/√3), sin30°=(1/2), cot45°=1 & sec60°=2, putting these values in the given expression,

we get:- $\left(cosec^2 45^\circ \sec^2 30^\circ\right)\left(\sin^2 30^\circ + 4\cot^2 45^\circ - \sec^2 60^\circ\right)$

$$= \left[(\sqrt{2})^2 \times \left(\frac{2}{\sqrt{3}}\right)^2\right]\left[\left(\frac{1}{2}\right)^2 + 4 \times (1)^2 - (2)^2\right]$$
$$= \left[2 \times \frac{4}{3}\right]\left[\frac{1}{4} + 4 - 4\right]$$
$$= \frac{8}{3} \times \frac{1}{4}$$
$$= \frac{2}{3}$$

10. Given, $\cos\theta = \frac{7}{25} \Rightarrow \cos^2\theta = \frac{49}{625}$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta = 1 - \frac{49}{625} = \frac{576}{625}$

$\Rightarrow \sin\theta = \frac{24}{25}$

Now, $\tan\theta + \cot\theta$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{1}{\sin\theta\cos\theta}$$
$$= \frac{1}{\frac{24}{25} \times \frac{7}{25}}$$
$$= \frac{625}{168}$$

11. According to the question, $\theta = 30°$

L.H.S.= $\cos 3\theta$

Putting $\theta = 30°$, we get

$= \cos 3 \times 30° = \cos 90° = 0$ ...(i)

R.H.S.= $4\cos^3\theta - 3\cos\theta$

Putting $\theta = 30°$, we get

$= 4\cos^3 30° - 3\cos 30°$

$$= 4\left(\frac{\sqrt{3}}{2}\right)^3 - 3 \times \frac{\sqrt{3}}{2}$$
$$= \frac{4 \times 3\sqrt{3}}{8} - \frac{3\sqrt{3}}{2}$$
$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0 \text{ ...(ii)}$$

From (i) and (ii)

L.H.S = R.H.S.

$\Rightarrow \cos 3\theta = 4\cos^3\theta - 3\cos\theta$

12. A = 60°, B = 30°

LHS = cos (A - B) = cos (60° - 30°) = cos 30° $= \frac{\sqrt{3}}{2}$

RHS = cos A cos B + sin A sin B

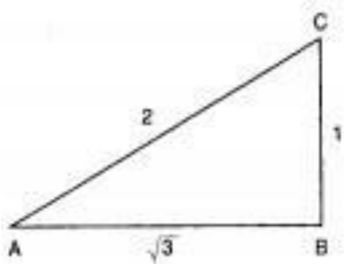
= cos 60° cos 30° + sin 60° sin 30°

$= \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$
$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$
$= \frac{2\sqrt{3}}{4}$
$= \frac{\sqrt{3}}{2}$

LHS = RHS Hence verified.

13. $cos\theta + sin\theta \ = \sqrt{2}cos\theta$

$\Rightarrow sin\theta = \sqrt{2}cos\theta - cos\theta$
$\Rightarrow sin\theta = (\sqrt{2} - 1)cos\theta$
$\Rightarrow \frac{\sin\theta}{\sqrt{2}-1}$ = cos θ
$\Rightarrow \frac{\sin\theta(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$ = cos θ
$\Rightarrow \sqrt{2}$ sin θ + sin θ = cos θ
$\Rightarrow$ cos θ - sin θ = $\sqrt{2}$ sin θ = RHS

14. 

According to the question,

$\cos ecA = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{2}{1}$

So, we draw a right triangle, right angled at B such that

Perpendicular $= BC = 1$, Hypotenuse $= AC = 2$

Using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$
$\Rightarrow \quad 2^2 = 1^2 + AB^2$
$\Rightarrow \quad 4 - 1 = AB^2$
$\Rightarrow \quad AB = \sqrt{3}$

$\tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$, $\sin A = \frac{BC}{AC} = \frac{1}{2}$ and $\cos A = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$

$\therefore \frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = \frac{1}{\frac{1}{\sqrt{3}}} + \frac{1/2}{1+\frac{\sqrt{3}}{2}}$

$\Rightarrow \frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = \frac{\sqrt{3}}{1} + \frac{1/2}{\frac{2+\sqrt{3}}{2}} = \frac{\sqrt{3}}{1} + \frac{1}{2+\sqrt{3}}$

$\Rightarrow \quad \frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = \sqrt{3} + \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$\Rightarrow \quad \frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = \sqrt{3} + \frac{2-\sqrt{3}}{2^2-(\sqrt{3})^2} = \sqrt{3} + \frac{2-\sqrt{3}}{4-3} = \sqrt{3} + (2-\sqrt{3}) = 2$

15. We have, $a\cos\theta - b\sin\theta = x$ ...(i)

and $a\sin\theta + b\cos\theta = y$ ...(ii)

Squaring Eq. (i) and (ii) and then adding, we get

$x^2 + y^2 = (a\cos\theta - b\sin\theta)^2 + (a\sin\theta + b\cos\theta)^2$

$\Rightarrow x^2 + y^2 = a^2\cos^2\theta + b^2\sin^2\theta$

$-2ab\cos\theta\sin\theta + a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta$ [$\because$ $(a + b)^2 = a^2 + b^2 + 2ab$ and $(a - b)^2 = a^2 + b^2 - 2ab$]

$\Rightarrow x^2 + y^2 = a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2\theta$

$\Rightarrow x^2 + y^2 = a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) [\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow x^2 + y^2 = a^2 + b^2$

Hence proved, LHS = RHS

16. Given,

$4\left(\frac{\sec^2 59^\circ - \cot^2 31^\circ}{3}\right) - \frac{2}{3}$ sin 90° + 3 $\tan^2 56° \times \tan^2 34° = \frac{x}{3}$

$\Rightarrow \frac{x}{3} = 4\left(\frac{cosec^2 31^\circ - \cot^2 31^\circ}{3}\right) - \frac{2}{3}$ + 3$\tan^2$ (90°-34°) × $\tan^2$ 34° [Since, Sec 59° = Sec(90° - 31°) = Cosec 31°]

$\Rightarrow \frac{x}{3} = 4\left(\frac{cosec^2 31^\circ - \cot^2 31^\circ}{3}\right) - \frac{2}{3}$ + 3 $\cot^2$ 34° × $\tan^2$ 34° [Since, tan(90° - A) = Cot A]

$\Rightarrow \frac{x}{3} = 4\left(\frac{1}{3}\right) - \frac{2}{3}$ + 3 [Since, $Cosec^2 A - Cot^2 A = 1$ & tan A .Cot A = 1]

$\Rightarrow \frac{x}{3} = \frac{4}{3} - \frac{2}{3} + 3$

$\Rightarrow \frac{x}{3} = \frac{2}{3} + 3 = \frac{11}{3}$

Hence, x = 11

17. In ΔABC, by angle sum property

A + B + C = 180°

$\Rightarrow$ B + C =180° - A .... (1)

Now , L.H.S

$\sin\frac{B+C}{2}$
$=\sin\left(\frac{180^\circ-A}{2}\right)$ [From (1)]
$=\sin\left(\frac{180^\circ}{2}-\frac{A}{2}\right)$
$=\sin\left(90^\circ-\frac{A}{2}\right)$ $[\sin(90^\circ-\theta)=\cos\theta]$
$=\cos\frac{A}{2}=RHS$ Hence Proved.

18. Let cot θ= x,
Then, $\sqrt{3}\cot^2\theta-4\cot\theta+\sqrt{3}=0$
$or, \sqrt{3}x^2\ -4x+\sqrt{3}=0$
$\sqrt{3}x^2\ -3x-x+\sqrt{3}=0$
$\sqrt{3}x(x-\sqrt{3})-1(x-\sqrt{3})=0$
$(x-\sqrt{3})(\sqrt{3}x-1)=0$
$\therefore\quad x=\sqrt{3}$ or $\frac{1}{\sqrt{3}}$
or, $\cot\theta=\sqrt{3}$ or $\cot\theta=\frac{1}{\sqrt{3}}$
$\therefore\quad \theta=30^\circ$ or $\theta=60^\circ$
When $\theta=30^\circ$
$\cot^2 30^\circ+\tan^2 30^\circ=(\sqrt{3})^2+\left(\frac{1}{\sqrt{3}}\right)^2$
$=3+\frac{1}{3}=\frac{10}{3}$
When θ = 60°,
$\cot^2 60° + \tan^2 60°$
= $\left(\frac{1}{\sqrt{3}}\right)^2+(\sqrt{3})^2$
$=\frac{1}{3}+3=\frac{10}{3}$

19. Given, $\tan\theta=\frac{2mn}{m^2-n^2}$ ......(1)
We know that,
$\sec^2\theta = 1 + \tan^2\theta$
= $1+\frac{(2mn)^2}{(m^2-n^2)^2}$ [from (1)]
$=\frac{(m^2-n^2)^2+4m^2n^2}{(m^2-n^2)^2}$
$\Rightarrow \sec^2\theta=\frac{(m^2+n^2)^2}{(m^2-n^2)^2}$

$\Rightarrow$ sec θ = $\sqrt{\frac{(m^2+n^2)^2}{(m^2-n^2)^2}}$

$\Rightarrow$ sec θ = $\frac{m^2+n^2}{m^2-n^2}$.........(2)

Now,

cos θ = $\frac{1}{\sec\theta} = \frac{m^2-n^2}{m^2+n^2}$ [from (2)]

cot θ = $\frac{1}{\tan\theta} = \frac{m^2-n^2}{2mn}$ [from (1)]

Also, tan θ = $\frac{\sin\theta}{\cos\theta}$

$\Rightarrow$ tan θ . cos θ = sin θ

$\Rightarrow \frac{2mn}{m^2-n^2} \times \frac{m^2-n^2}{m^2+n^2}$ = sin θ

$\Rightarrow$ sin θ = $\frac{2mn}{m^2+n^2}$

& , cosec θ = $\frac{1}{\sin\theta} = \frac{m^2+n^2}{2mn}$

20. In $\triangle PQR, \quad \therefore \angle Q = 90^\circ$

$\therefore$ $PR^2 = PQ^2 + QR^2$............ By Pythagoras theorem

$\Rightarrow (13)^2 = (12)^2 + QR^2$

$\Rightarrow 169 = 144 + QR^2$

$\Rightarrow QR^2 = 169 - 144 \Rightarrow QR^2 = 25$

$\Rightarrow QR = \sqrt{25} = 5\text{cm}$

$\therefore \tan P - \cot R = \frac{QR}{PQ} - \frac{QR}{PQ} = 0$