

CBSE Test Paper 01
Chapter 2 Polynomials

1. The zeroes of a polynomial $x^2 + 5x - 24$ are **(1)**
 - a. one positive and one negative
 - b. both positive
 - c. both negative
 - d. both equal
2. If ' α ' and ' β ' are the zeroes of a quadratic polynomial $x^2 - 5x + b$ and $\alpha - \beta = 1$, then the value of 'b' is **(1)**
 - a. -6
 - b. -5
 - c. 5
 - d. 6
3. Degree of the polynomial $2x^4 + 3x^3 - 5x^2 + 9x + 1$ is **(1)**
 - a. 3
 - b. 1
 - c. 2
 - d. 4
4. If α and β are zeros of $x^2 + 5x + 8$, then the value of $(\alpha + \beta)$ is **(1)**
 - a. -8
 - b. 8
 - c. 5
 - d. -5
5. Which of the following expressions is not a polynomial? **(1)**
 - a. $5x^3 - 3x^2 - \sqrt{x} + 2$
 - b. $5x^3 - 3x^2 - x + \sqrt{2}$
 - c. $5x^2 - \frac{2}{3}x + 2\sqrt{5}$
 - d. $\sqrt{5}x^3 - \frac{3}{5}x + \frac{1}{7}$
6. Find the zeroes of the polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$. **(1)**
7. If the product of the zeros of the polynomial $(ax^2 - 6x - 6)$ is 4. Find the value of a. **(1)**
8. If $x^3 + x^2 - ax + b$ is divisible by $(x^2 - x)$, write the values of a and b. **(1)**

9. Find the zeros of the following quadratic polynomial and verify the relationship between the zeros and the coefficients: $3x^2 - x - 4$. **(1)**
10. Find all the zeroes of $f(x) = x^2 - 2x$. **(1)**
11. If α and β are the zeroes of the polynomial $4x^2 - 2x + (k - 4)$ and $\alpha = \frac{1}{\beta}$, find the value of k . **(2)**
12. If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is the reciprocal of the other, find the value of a . **(2)**
13. Find a cubic polynomial whose zeros are $3, \frac{1}{2}$ and -1 . **(2)**
14. If α and β are zeroes of the polynomial $x^2 - p(x + 1) + c$ such that $(\alpha + 1)(\beta + 1) = 0$, then find the value of c . **(3)**
15. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a . **(3)**
16. Find the zeroes of the given quadratic polynomials and verify the relationship between the zeroes and the coefficients. $x^2 - 2x - 8$ **(3)**
17. A polynomial $g(x)$ of degree zero is added to polynomial $2x^5 + 5x^4 - 14x + 10$, so that it becomes exactly divisible by $2x - 3$. Find $g(x)$. **(3)**
18. A village of the North-East India is suffering from flood. A group of students decide to help them with food items, clothes etc, So the student collects some amount of rupees, which is represented by $x^4 + x^3 + 8x^2 + ax + b$
- If the number of students is represented by $x^2 + 1$, find the values of a and b .
 - What values have been depicted by the group of students? **(4)**
19. If α and β are the zeroes of the polynomial $p(x) = 6x^2 + 5x - k$ satisfying the relation, $\alpha - \beta = \frac{1}{6}$, then find the value of k . **(4)**
20. If α and β are the zeroes of polynomial $p(x) = 3x^2 + 2x + 1$, find the polynomial whose zeroes are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$. **(4)**

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Solution

1. a. one positive and one negative

Explanation: $x^2 + 5x - 24$
 $= x^2 + 8x - 3x - 24$
 $= x(x + 8) - 3(x + 8) = 0$
 $(x + 8)(x - 3) = 0$
 $\therefore x + 8 = 0$ or $x - 3 = 0$
 $\Rightarrow x = -8$ or $x = 3$

2. d. 6

Explanation: Here $\alpha + \beta = \frac{-b}{a} = \frac{-(-5)}{1} \alpha + \beta = 5$ (i)

And it is given that $\alpha - \beta = 1$ (ii)

On solving eq. (i) and eq. (ii), we get

$$\alpha + \beta = 5$$

$$\alpha - \beta = 1$$

$$2\alpha = 6 \text{ (}\beta \text{ is cancelled)}$$

$$\alpha = \frac{6}{2}$$

$$\alpha = 3 \text{ Put the value of } \alpha \text{ in eq. (i)}$$

$$\alpha + \beta = 5$$

$$\Rightarrow 3 + \beta = 5$$

$$\Rightarrow \beta = 5 - 3$$

$$\Rightarrow \beta = 2$$

$$\therefore \alpha\beta = \frac{c}{a}$$

$$\Rightarrow 3 \times 2 = \frac{b}{1} \Rightarrow b = 6$$

3. d. 4

Explanation: The highest power of the variable is 4. So, the degree of the polynomial is 4.

4. d. -5

Explanation: $x^2 + 5x + 8$

$$\begin{aligned}\alpha + \beta &= \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} \\ &= \frac{-5}{1} \\ &= -5\end{aligned}$$

5. a. $5x^3 - 3x^2 - \sqrt{x} + 2$

Explanation: $5x^3 - 3x^2 - \sqrt{x} + 2$ is not a polynomial because each term of a polynomial should be a product of a constant and one or more variable raised to a positive, zero or integral power. Here \sqrt{x} does not satisfy the condition of being a polynomial.

6. We have to find the zeroes of the polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$.

$$\begin{aligned}p(x) &= \sqrt{3}x^2 - 8x + 4\sqrt{3} \\ &= \sqrt{3}x^2 - 6x - 2x + 4\sqrt{3} = 0 \\ &= \sqrt{3}(x - 2\sqrt{3}) - 2(x - 2\sqrt{3}) \\ &= (\sqrt{3}x - 2)(x - 2\sqrt{3}) = 0 \\ \therefore \text{Zeroes} &= \frac{2}{\sqrt{3}}, 2\sqrt{3}\end{aligned}$$

7. According to the question, we have to find the value of a such that the product of the zeros of the polynomial $(ax^2 - 6x - 6)$ is 4.

Let α and β be the zeros of the polynomial $(ax^2 - 6x - 6)$

$$\text{Then, } \alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{-6}{a}$$

But, $\alpha\beta = 4$ (given).

$$\therefore \frac{-6}{a} = 4 \Rightarrow 4a = -6 \Rightarrow a = \frac{-6}{4} = \frac{-3}{2}$$

$$\text{Hence, } a = \frac{-3}{2}$$

8. Since $f(x) = x^3 + x^2 - ax + b$ is divisible by $(x^2 - x)$, we have

$$x^2 - x = 0$$

$$\Rightarrow x(x - 1) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 1$$

Hence,

$$f(0) = 0$$

$$\Rightarrow x^3 + x^2 - ax + b = 0$$

$$\Rightarrow 0^3 + 0^2 - a(0) + b = 0$$

$$\Rightarrow b = 0$$

Also,

$$f(1) = 0$$

$$\Rightarrow x^3 + x^2 - ax + b = 0$$

$$\Rightarrow 1^3 + 1^2 - a(1) + 0 = 0$$

$$\Rightarrow 1 + 1 - a = 0$$

$$\Rightarrow 2 - a = 0$$

$$\Rightarrow a = 2$$

Hence, the value of a and b in given polynomial are $a = 2$ and $b = 0$.

9. We have, $f(x) = 3x^2 - x - 4$

$$= 3x^2 - 4x + 3x - 4$$

$$= x(3x - 4) + 1(3x - 4)$$

$$= (3x - 4)(x + 1)$$

$$\therefore f(x) = 0$$

$$\Rightarrow (3x - 4)(x + 1) = 0$$

$$\Rightarrow 3x - 4 = 0 \text{ or } x + 1 = 0$$

$$\Rightarrow x = \frac{4}{3} \text{ or } x = -1$$

So, the zeros of $f(x)$ are $\frac{4}{3}$ and -1

$$\text{Now sum of zeros} = \frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{And product of zeros} = \frac{4}{3} \times (-1) = \frac{-4}{3} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

10. $f(x) = x^2 - 2x$

$$= x(x - 2)$$

$$f(x) = 0 \Rightarrow x = 0 \text{ or } x = 2$$

Hence, zeroes are 0 and 2.

11. Here $p(x) = 4x^2 - 2x + k - 4$

$$\text{Here } a=4, b=-2, c=k-4$$

Given α and β are zeros of the given polynomial

$$\alpha = \frac{1}{\beta},$$

$$\Rightarrow \beta = \frac{1}{\alpha}$$

$$\alpha\beta = 1$$

$$\text{also } \alpha\beta = \frac{c}{a} = \frac{k-4}{4}$$

$$\text{So } \frac{k-4}{4} = 1$$

$$k - 4 = 4$$

$$k = 4 + 4 = 8$$

12. Let α and $\frac{1}{\alpha}$ be the zeros of $(a^2 + 9)x^2 + 13x + 6a$.

Then, we have

$$\alpha \times \frac{1}{\alpha} = \frac{6a}{a^2+9}$$

$$\Rightarrow 1 = \frac{6a}{a^2+9}$$

$$\Rightarrow a^2 + 9 = 6a$$

$$\Rightarrow a^2 - 6a + 9 = 0$$

$$\Rightarrow a^2 - 3a - 3a + 9 = 0$$

$$\Rightarrow a(a - 3) - 3(a - 3) = 0$$

$$\Rightarrow (a - 3)(a - 3) = 0$$

$$\Rightarrow (a - 3)^2 = 0$$

$$\Rightarrow a - 3 = 0$$

$$\Rightarrow a = 3$$

So, the value of a in given polynomial is 3.

13. Let $\alpha = 3, \beta = \frac{1}{2}$ and $\gamma = -1$. Then,

$$(\alpha + \beta + \gamma) = \left(3 + \frac{1}{2} - 1\right) = \frac{5}{2},$$

$$(\alpha\beta + \beta\gamma + \gamma\alpha) = \left(\frac{3}{2} - \frac{1}{2} - 3\right) = \frac{-4}{2} = -2$$

$$\text{and } \alpha\beta\gamma = \left\{3 \times \frac{1}{2} \times (-1)\right\} = \frac{-3}{2}$$

The polynomial with zeros α, β and γ is:

$$\begin{aligned} & x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma \\ &= x^3 - \frac{5}{2}x^2 - 2x + \frac{3}{2} \end{aligned}$$

Thus, $2x^3 - 5x^2 - 4x + 3$ is the desired polynomial.

14. Given, α and β are the zeroes of polynomial $x^2 - p(x + 1) + c$

which can be written as $x^2 - px + c - p$

So, sum of zeroes, $\alpha + \beta = p$ [\because sum of coefficients = $\frac{-(\text{coefficient}(x))}{\text{coefficient}(x^2)}$]

and product of zeroes $\alpha\beta = c - p$ [\because product of coefficients = $\frac{\text{constant_term}}{\text{coefficient}(x^2)}$],

$$\text{Also, } (\alpha + 1)(\beta + 1) = 0$$

$$\alpha\beta + \alpha + \beta + 1 = 0$$

$$\Rightarrow c - p + p + 1 = 0$$

$$\Rightarrow c = -1$$

15. On dividing $x^4 - 6x^3 - 16x^2 - 25x + 10$ by $x^2 - 2x + k$

$$\begin{array}{r}
 x^2 - 4x + (8 - k) \\
 x^2 - 2x + k \overline{) x^4 - 6x^3 + 16x^2 - 25x + 10} \\
 \underline{x^4 - 2x^3 + kx^2} \\
 -4x^3 + (16 - k)x^2 - 25x + 10 \\
 \underline{-4x^3 + 8x^2 - 4kx} \\
 + - + \\
 (8 - k)x^2 + (4k - 25)x + 10 \\
 \underline{(8 - k)x^2 - 2(8 - k)x + (8 - k)k} \\
 - + - \\
 (2k - 9)x - (8 - k)k + 10
 \end{array}$$

∴ Remainder = $(2k - 9)x - (8 - k)k + 10$

But the remainder is given as $x + a$.

On comparing their coefficients,

$$2k - 9 = 1$$

$$\Rightarrow k = 10$$

$$\Rightarrow k = 5 \text{ and,}$$

$$-(8 - k)k + 10 = a$$

$$\Rightarrow a = -(8 - 5)5 + 10 = -15 + 10 = -5$$

Hence, $k = 5$ and $a = -5$

16. Let $p(x) = x^2 - 2x - 8$

By the method of splitting the middle term,

$$x^2 - 2x - 8 = x^2 - 4x + 2x - 8$$

$$= x(x - 4) + 2(x - 4) = (x - 4)(x + 2)$$

For zeroes of $p(x)$,

$$p(x) = 0$$

$$\Rightarrow (x - 4)(x + 2) = 0$$

$$\Rightarrow x - 4 = 0 \text{ or } x + 2 = 0$$

$$\Rightarrow x = 4 \text{ or } x = -2$$

$$\Rightarrow x = 4, -2$$

So, the zeroes of $p(x)$ are 4 and -2.

We observe that, Sum of its zeroes

$$= 4 + (-2) = 2$$

$$= \frac{-(-2)}{1} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

Product of its zeroes

$$= 4x(-2) = -8 = \frac{-8}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Hence, relation between zeroes and coefficients is verified.

17. According to the question, $g(x)$ of degree zero is added to the polynomial $(2x^3 + 2x^2 - 14x + 10)$

such that it becomes completely divisible by $2x - 3$.

Let $g(x)=k$, then $2x^3 + 5x^2 - 14x + 10 + k$ will be exactly divisible by $2x - 3$.

$$P(x) = 2x^3 + 5x^2 - 14x + 10 + k$$

We know dividend = quotient \times divisor + remainder

On dividing $2x^3 + 5x^2 - 14x + 10 + k$ by $2x - 3$, we get quotient $x^2 + 4x - 1$ and remainder = $k+7$

The degree of $g(x)$ is zero then $g(x) = 0$

$$\Rightarrow k + 7 = 0 \Rightarrow k = -7$$

$$\therefore g(x) = -7$$

18. i. First we divide $x^4 + x^3 + 8x^2 + ax + b$ by $x^2 + 1$ as follows:

$$\begin{array}{r} x^2 + x + 7 \\ x^2 + 1 \overline{) x^4 + x^3 + 8x^2 + ax + b} \\ \underline{x^4 + x^2} \\ x^3 + 7x^2 + ax + b \\ \underline{x^3 + x} \\ 7x^2 + (a-1)x + b \\ \underline{7x^2 + 7} \\ (a-1)x + (b-7) \end{array}$$

Since $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$, therefore remainder = 0

$$\text{i.e. } (a-1)x + (b-7) = 0$$

$$\text{or } (a-1)x + (b-7) = 0x + 0$$

Equating the corresponding terms, We have

$$a - 1 = 0 \text{ and } b - 7 = 0$$

$$\text{i.e. } a = 1 \text{ and } b = 7$$

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19. According to the question, α and β are zeroes of $p(x) = 6x^2 - 5x + k$

$$\text{So, Sum of zeroes} = \alpha + \beta = \dots = \left(\frac{-5}{6} \right) = \frac{5}{6} \dots \dots \dots \text{(i)}$$

$$\alpha - \beta = \frac{1}{6} \text{ (Given) (ii)}$$

Adding equations (i) and (ii) , we get

$$2\alpha = 1$$

$$\text{or, } \alpha = \frac{1}{2}$$

On putting the value of α in equation (ii), we get

$$\frac{1}{2} - \beta = \frac{1}{6}$$

$$\beta = \frac{1}{2} - \frac{1}{6}$$

$$\beta = \frac{2}{6} = \frac{1}{3}$$

$$\therefore \alpha\beta = \frac{k}{6} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

Hence, $k = 1$

20. Since α and β are the zeroes of polynomial $3x^2 + 2x + 1$.

$$\text{Hence, } \alpha + \beta = -\frac{2}{3}$$

$$\text{and } \alpha\beta = \frac{1}{3}$$

Now, for the new polynomial,

$$\text{Sum of zeroes} = \frac{1-\alpha}{1+\alpha} + \frac{1-\beta}{1+\beta}$$

$$= \frac{(1-\alpha+\beta-\alpha\beta)+(1+\alpha-\beta-\alpha\beta)}{(1+\alpha)(1+\beta)}$$

$$= \frac{2-2\alpha\beta}{1+\alpha+\beta+\alpha\beta} = \frac{2-\frac{2}{3}}{1-\frac{2}{3}+\frac{1}{3}}$$

$$\therefore \text{Sum of zeroes} = \frac{4/3}{2/3} = 2$$

$$\text{Product of zeroes} = \left[\frac{1-\alpha}{1+\alpha} \right] \left[\frac{1-\beta}{1+\beta} \right]$$

$$= \frac{(1-\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$$

$$= \frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$$

$$= \frac{1+\frac{2}{3}+\frac{1}{3}}{1-\frac{2}{3}+\frac{1}{3}} = \frac{\frac{6}{3}}{\frac{3}{3}} = 3$$

Hence, Required polynomial = $x^2 - (\text{Sum of zeroes})x + \text{Product of zeroes}$

$$= x^2 - 2x + 3$$