CBSE Test Paper 03

Chapter 2 Polynomials

1. If the polynomial $3x^3 - 4x^2 - 17x - k$ is exactly divisible by x - 3, then the value of 'k' is **(1)**
   a. 6
   b. 5
   c. -5
   d. -6
2. The number of zeroes that the polynomial $f(x) = (x - 2)^2 + 4$ can have is **(1)**
   a. 0
   b. 2
   c. 3
   d. 1
3. If '2' is the zero of both the polynomials $3x^2 + mx - 14$ and $2x^3 + nx^2 + x - 2$, then the value of m – 2n is **(1)**
   a. 5
   b. -1
   c. 9
   d. -9
4. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is –1, then the product of the other two zeroes is **(1)**
   a. $b - a + 1$
   b. $b + a + 1$
   c. $b + a - 1$
   d. $b - a - 1$
5. A quadratic polynomial with zeroes $\frac{1}{4}$ and – 1 is **(1)**
   a. $4x^2 + 3x - 1$
   b. $4x^2 - 3x - 1$
   c. $4x^2 - 3x + 1$
   d. $4x^2 + 3x + 1$
6. If(a-b), a and (a+b) are zeros of the polynomial $2x^3-6x^2+5x-7$, write the value of a. **(1)**

7. The sum of the zeros and the product of zeros of a quadratic polynomial are $\frac{-1}{2}$ and -3 respectively. Write the polynomial. **(1)**
8. Find the quadratic polynomial whose zeroes are $\sqrt{3}+\sqrt{5}$ and $\sqrt{5}-\sqrt{3}$ **(1)**
9. Sum and product of zeroes of quadratic polynomial are 5 and 17 respectively. Find the polynomial. **(1)**
10. Find a quadratic polynomial whose one zero is -8 and sum of zeroes is 0. **(1)**
11. Find the zeros of the quadratic polynomial $f(x) = abx^2 + (b^2 - ac)x - bc$, and verify the relationship between the zeros and its coefficients. **(2)**
12. Find the zeros of the following quadratic polynomials and verify the relationship between the zeros and the coefficients: $8x^2 - 4$ **(2)**
13. Find a quadratic polynomial whose zeroes are $5 + \sqrt{2}$ and $5 - \sqrt{2}$. **(2)**
14. Find a quadratic polynomial of the given number as the sum and product of its zeroes respectively. 0, $\sqrt{7}$ **(3)**
15. Find all the zeroes of $p(x) = x^3 - 9x^2 - 12x + 20$ if $x + 2$ is a factor of $p(x)$. **(3)**
16. If α and β are the zeroes of the quadratic polynomial $f(x) = 6x^2 + x - 2$, then, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$. **(3)**
17. Can $(x - 2)$ be the remainder on division of a polynomial $p(x)$ by $(2x + 3)$? Justify your answer. **(3)**
18. α, β and γ are zeroes of the polynomial $x^3 + px^2 + qx + 2$ such that $\alpha.\beta + 1 = 0$. Find the value of $2p + q + 5$. **(4)**
19. Obtain all other zeroes of the polynomial $x^4 + 6x^3 + x^2 - 24x - 20$, if two of its zeroes are + 2 and - 5. **(4)**
20. When a polynomial $f(x)$ is divided by $x^2 - 5$, the quotient is $x^2 - 2x - 3$ and remainder is zero. Find the polynomial and all its zeroes. **(4)**

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Solution

1. d. -6

   Explanation: If the polynomial $3x^3 - 4x^2 - 17x - k$ is exactly divisible -- $x -$ - 3then

   $p(3) = 0$ (By factor theorem)

   $\Rightarrow 3(3)^3 - 4(3)^2 - 17 \times 3 - k = 0$

   $\Rightarrow 81 - 36 - 51 - k = 0$

   $\Rightarrow -6 - k = 0$

   $\Rightarrow k = -6$

2. b. 2

   Explanation: $f(x) = (x - 2)^2 + 4 = x^2 - 4x + 4 + 4 = x^2 - 4x + 8$

   Here the largest exponent of variable is 2,

   therefore number of zeroes of the given polynomial is 2.

3. c. 9

   Explanation: According to the question, $p(2) = 3x^2 + mx - 14 = 0$

   $\Rightarrow 3(2)^2 + m \times 2 - 14 = 0$

   $\Rightarrow 12 + 2m - 14 = 0 \Rightarrow m = 1$

   Also $p(2) = 2x^3 + nx^2 + x - 2 = 0$

   $\Rightarrow 2 \times (2)^3 + n \times (2)^2 + 2 - 2 = 0$

   $\Rightarrow 16 + 4n = 0$

   $\Rightarrow n = -4$

   $\therefore m - 2n = 1 - 2 \times (-4) = 1 + 8 = 9$

4. a. $b - a + 1$

   Explanation: Let α, β, γ are the zeroes of the given polynomial. Given: $\alpha = 1$ and To find : $\beta\gamma$ Since, $\alpha + \beta + \gamma = \frac{-b}{a} \Rightarrow -1 + \beta + \gamma = \frac{-a}{1} \Rightarrow$

   $\beta + \gamma = -a + 1$ ....(i)

   Also $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} \Rightarrow (-1)\beta + \beta\gamma + (-1)\gamma = \frac{b}{1}$

   $\Rightarrow -\beta + \beta\gamma - \gamma = b \Rightarrow \beta\gamma - (\beta + \gamma) = b \Rightarrow \beta\gamma - (-a + 1) = b$ [From eq. (i)]

   $\Rightarrow \beta\gamma = b - a + 1$

5. a. $4x^2+3x-1$

Explanation: x^2-($\alpha+\beta$)x + $(\alpha\beta)$ =0

Here α = $=\frac{1}{4}$ $(-1)=\frac{1-4}{4}=\frac{-3}{4}+\beta$ $=+\frac{1}{4}\times(-1)=\frac{-1}{4}$ And $\alpha\beta$ $\frac{c}{a}$

x^2 -($\frac{-3}{4}$)x + ($\frac{-1}{4}$) = 0

x^2 + $\frac{3}{4}$ x - $\frac{1}{4}$ = 0

$\frac{4x^2+3x-1}{4}$ = 0 (By L.C.M)

$4x^2+3x-1=0$

6. Given polynomial is p(x) = $2x^3 - 6x^2 + 5x - 7$

Let α = (a - b), β = a and γ = (a + b)

Now, $\alpha+\beta+\gamma$ = $-\frac{(-6)}{2}$ = 3

⇒ (a - b) + a + (a + b) = 3

⇒ a - b + a + a + b = 3

⇒ 3a = 3

⇒ a = 3/3

⇒ a = 1

So the value of a in given polynomial is 1.

7. Let α and β be the zeros of the required quadratic polynomial.

As per given condition the sum of the zeros and the product of zeros of a quadratic polynomial are $\frac{-1}{2}$ and -3 respectively.

Then, we have

$\alpha+\beta=-\frac{1}{2}$ and $\alpha\beta$ = -3

Now, a quadratic polynomial whose zeros are α and β is given by

p(x) = x^2 - $(\alpha+\beta)x$ + $\alpha\beta$

∴ Required quadratic polynomial,

p(x) = x^2 - $(-\frac{1}{2})$ x + (-3)

= x^2 + $\frac{1}{2}$ x - 3

Hence the given polynomial is x^2 + $\frac{1}{2}$ x - 3 .

8. Sum of zeroes = $\sqrt{3}+\sqrt{5}+\sqrt{5}-\sqrt{3}=2\sqrt{5}$

Product of zeroes = $(\sqrt{3}+\sqrt{5})(\sqrt{5}-\sqrt{3})=\sqrt{15}-\sqrt{9}+\sqrt{25}-\sqrt{15}$

$=\sqrt{15}-3+5-\sqrt{15}$

$= -3 + 5 = 2$

Quadratic polynomial is x^2-(sum of zeroes)x + product of zeroes

= $x^2 - 2\sqrt{5}x + 2$

9. Sum of zeroes = 5

   Product of zeroes = 17

   Quadratic Polynomial = x^2 - (sum of zeroes)$x+$ product of zeroes = $x^2 - 5x + 17$

10. It is given that One zero = - 8

    and Sum of zeroes = 0

    Since sum of zeroes $= \alpha + \beta$

    $\therefore$ Other zero = 0 - (-8) = 8

    Product of zeroes = 8 $\times$ (- 8) = - 64

    Hence, Polynomial p(x) = x^2 - (S)x + P

    = x^2 - 64

11. We have,

    f(x) = $abx^2 + (b^2 - ac)x - bc$

    = $abx^2 + b^2x - acx - bc$

    = bx(ax + b) - c(ax + b)

    = (ax + b) (bx - c)

    Now r(x)=0 if

    ax+b=0 or bx-c=0

    $i.e. X = -\frac{b}{a} \ or \ X = \frac{c}{b}$

    Thus, the zeroes of f(x) are :

    $\alpha = -\frac{b}{a} \ and \ \beta = \frac{c}{b}$

    $\alpha + \beta = -\frac{\mathrm{b}}{\mathrm{a}} + \frac{\mathrm{c}}{\mathrm{b}} = \frac{-\mathrm{b}^2 + ac}{ab} = -\frac{(b^2 - ac)}{ab} - (1)$

    $\alpha\beta = \frac{\mathrm{b}}{\mathrm{a}} \times -\frac{\mathrm{c}}{\mathrm{b}} = -\frac{\mathrm{c}}{\mathrm{a}} \dots . (2)$

    Now for f(x) = $abx^2 + (b^2 - ac)x - bc$

    A = ab , B= b^2 - ac, C = -b

    $-\frac{B}{A} = -\frac{b^2 - ac}{ab} - -(3)$

    $\frac{C}{A} = \frac{-bc}{ab} = -\frac{c}{a} - - - -(4)$

    From (1) & (3) and (2) & (4) we conclude:

$\alpha+\beta=-\frac{B}{A}$
$\alpha\beta=\frac{C}{\text{A}}$

12. f(x) = $8x^2$ - 4

= 4 ($2x^2$ - 1)
$=4[(\sqrt{2}x)^2-1^2)$
$=4(\sqrt{2}x-1)(\sqrt{2}x+1)$
$f(x)=0\Rightarrow(\sqrt{2}x-1)(\sqrt{2}x+1)=0$
$\therefore\sqrt{2}x-1=0$ or $\sqrt{2}x+1=0$
$\therefore x=\frac{1}{\sqrt{2}}$ or $x=-\frac{1}{\sqrt{2}}$
So, the zeros of f(x) are $\frac{1}{\sqrt{2}}$ and $-\frac{1}{\sqrt{2}}$
Sum of zeros $=\left(\frac{1}{\sqrt{2}}\right)+\left(-\frac{1}{\sqrt{2}}\right)=0=\frac{0}{8}=\frac{\text{Coeff. of }x}{\text{coeff. of }x^2}$
Product of zeros = $\left(\frac{1}{\sqrt{2}}\right)\times\left(-\frac{1}{\sqrt{2}}\right)=-\frac{1}{2}=\frac{-4}{8}$
$=\frac{\text{constant term}}{\text{Coeff. of }x^2}$

13. Let α,β are zeroes of quadratic polynomial p(x).
$\therefore\quad p(x)=x^2-(\alpha+\beta)x+\alpha\beta$
Here, $\alpha=5+\sqrt{2},\beta=5-\sqrt{2}$
$\therefore\quad\alpha+\beta=5+\sqrt{2}+5-\sqrt{2}$ = 10
and $\alpha\beta=(5+\sqrt{2})(5-\sqrt{2})$ = 25 - 2 = 23

Hence the polynomial is p(x) = x^2 - 10x + 23.

14. 0, $\sqrt{7}$

Let the quadratic polynomial be ax^2 + bx + c

and its zeroes be α and β,

Then, $\alpha+\beta=0=-\frac{b}{a}$ and, $Q\beta=\sqrt{3}=\frac{c}{a}$

If a = 1, then b = 0 and $c=\sqrt{7}$

So, one quadratic polynomial which fits

the given conditions is $x^2+\sqrt{7}$

15. Given polynomial is p(x) = x^3 - $9x^2$ - 12x + 20 and x + 2 is a factor of p(x).

Quotient: $x^2 - 11x + 10$

$$x + 2 \,\overline{\big)\, x^3 - 9x^2 - 12x + 20}$$

$x^3 + 2x^2$

$-11x^2 - 12x$

$-11x^2 - 22x$

$10x + 20$

$10x + 20$

0

dividend = divisor × quotient + remainder

∴ p(x) = (x + 2) (x^2 - 11x + 10)

= (x + 2) (x^2 - 10x - x + 10)

= (x + 2) [x (x - 10) - 1 (x - 10)]

= (x + 2) (x - 1) (x - 10)

∴ Zeroes of p(x) are -2, 1, 10.

16. f(x) = $6x^2$ + x - 2

a = 6, b = 1, c = -2

Let zeroes be α and β.Then

Sum of zeroes= $\alpha + \beta = -\frac{b}{a} = -\frac{1}{6}$

Product of zeroes $\alpha \times \beta = \frac{c}{a} = \frac{-2}{6} = -\frac{1}{3}$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}\left[\because (\alpha+\beta)^2 = \alpha^2+\beta^2+2\alpha\beta\right]$$

$$= \frac{\left[-\frac{1}{6}\right]^2 - 2\left[-\frac{1}{3}\right]}{\left[-\frac{1}{3}\right]}$$

$$= \frac{\frac{1}{36}+\frac{2}{3}}{-\frac{1}{3}}$$

$$= \frac{\frac{1+24}{36}}{-\frac{1}{3}}$$

$$= \frac{25}{36} \times \frac{-3}{1}$$

$$= \frac{-25}{12}$$

17. No, (x – 2) cannot be remainder on division of polynomial p(x) by (2x + 3) because the degree of remainder is either 0 or its degree is less than the degree of the divisor.

18. P(x) = $x^3 + px^2 + qx + 2$

Here, a = 1, b = p, c = q, d = 2

Now, $\alpha + \beta + \gamma = \frac{-b}{a}$ = -p ....(i)

$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$ = q

$\Rightarrow \alpha\beta + \gamma(\beta + \alpha)$ = q ....(ii)

and $\alpha \cdot \beta \cdot \gamma = \frac{-d}{a}$ = -2

$\Rightarrow \quad \alpha. \beta \cdot \gamma$ = -2 ....(iii)

Also, $\alpha\beta + 1 = 0 \Rightarrow \alpha\beta = -1$

Therefore, (iii) becomes $-1 \times \gamma = -2 \Rightarrow \gamma = 2$

Substituting in (i), we get

$\alpha + \beta + 2 = -p \Rightarrow \alpha + \beta$ =- p - 2

Substituting these value in (ii), we get

-1 + 2(-p - 2) = q

$\Rightarrow$ -1 - 2p - 4 = q

$\Rightarrow$ 2p + q + 5 = 0

19. As x = 2 and -5 are the zeroes of $x^4 + 6x^3 + x^2 - 24x - 20$.

$\Rightarrow$ (x - 2) and (x + 5) are two factors of $x^4 + 6x^3 + x^2 - 24x - 20$

$\Rightarrow$ product of factors is (x - 2) (x + 5) = $x^2 + 3x - 10$

Dividing $x^4 + 6x^3 + x^2 - 24x - 20$ by $x^2 + 3x - 10$

$$
\begin{array}{r|l}
 & x^2 + 3x + 2 \\
x^2 + 3x - 10 & x^4 + 6x^3 + x^2 - 24x - 20 \\
 & x^4 + 3x^3 - 10x^2 \\
 & - \quad - \quad + \\
 & 3x^3 + 11x^2 - 24x - 20 \\
 & 3x^3 + 9x^2 - 30x \\
 & - \quad - \quad + \\
 & 2x^2 + 6x - 20 \\
 & 2x^2 + 6x - 20 \\
 & - \quad - \quad + \\
 & 0
\end{array}
$$

Dividend = divisor $\times$ quotient + remainder

$\Rightarrow x^4 + 6x^3 + x^2 - 24x - 20 = (x^2 + 3x - 10)(x^2 + 3x + 2)$

= (x - 2) (x + 5) (x + 2) (x + 1)

Hence, other two zeroes are -2 and -1.

20. $g(x) = x^2 - 5$

$q(x) = x^2 - 2x - 3$

$r(x) = 0$

By division algorithm for polynomials, we have

$f(x) = q(x).g(x) + r(x)$

$f(x) = (x^2 - 5)(x^2 - 2x - 3) + 0$

$f(x) = x^4 - 2x^3 - 3x^2 - 5x^2 + 10x + 15$

$f(x) = x^4 - 2x^3 - 8x^2 + 10x + 15$

So, the required polynomial is $f(x) = x^4 - 2x^3 - 8x^2 + 10x + 15$ Now,

q(x) and g(x) will be factors of f(x)

$x^2 - 5 = 0$ and $x^2 - 2x - 3 = 0$

$x^2 - (\sqrt{5})^2 = 0$ and $x^2 + x - 3x - 3 = 0$

$(x - \sqrt{5})(x + \sqrt{5}) = 0$ and $(x + 1)(x - 3) = 0$

$x = \sqrt{5}$, $x = -\sqrt{5}$, $x = -1$ and $x = 3$

So, the zeroes are $\alpha = \sqrt{5}$, $\beta = -\sqrt{5}$, $\gamma = -1$ and $\delta = 3$.