CBSE Test Paper 04
Chapter 2 Polynomials

1. If 2, – 7 and – 14 are the sum, sum of the product of its zeroes taken two at a time and the product of its zeroes of a cubic polynomial, then the cubic polynomial is **(1)**

   a. $x^3 - 2x^2 - 7x - 14$
   b. $x^3 + 2x^2 + 7x + 14$
   c. $x^3 - 2x^2 - 7x + 14$
   d. $x^3 - 2x^2 + 7x + 14$

2. A polynomial of degree_____is called a cubic polynomial. **(1)**

   a. 2
   b. 0
   c. 1
   d. 3

3. The number polynomials having zeroes as – 2 and 5 is **(1)**

   a. 1
   b. 2
   c. 3
   d. more than 3

4. A real number 'k' is said to be a zero of a polynomial p(x), if p(k) =. **(1)**

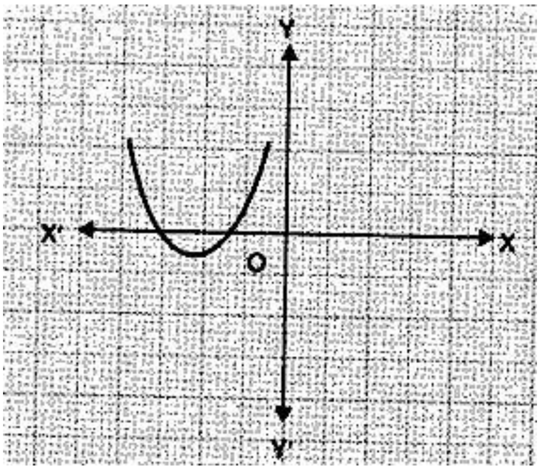
   a. 0
   b. 2
   c. 3
   d. 1

5. If 'α' and 'β' are the zeroes of a quadratic polynomial $x^2 + 5x - 5$, then **(1)**

   a. $\alpha + \beta = \alpha\beta$
   b. $\alpha - \beta = \alpha\beta$

c. $\alpha + \beta > \alpha\beta$
d. $\alpha + \beta < \alpha\beta$

6. Find the number of zeroes of p(x). The graph of y = p(x) is given in figure below, for some polynomial p(x): **(1)**



7. If α and β are the zeros of the quadratic polynomial f(x) = ax^2 +bx + c, then evaluate $\alpha^2 + \beta^2$. **(1)**

8. Find the zeros of the quadratic polynomial and check the relationship between the zeros and the coefficients. **(1)**

$4x^2 - 4x - 3 = 0$

9. If α and β are the zeroes of the quadratic polynomial f(x) = $ax^2 + bx + c$, then find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$. **(1)**

10. If p(x) = 5x - 10 is divided by x - $\sqrt{2}$, then find remainder. **(1)**

11. If α and β are the zeroes of a polynomial $x^2 - 4\sqrt{3}$ x + 3, then find the value of $\alpha + \beta - \alpha\beta$. **(2)**

12. Find all other zeroes of the polynomial $2x^3 - 4x - x^2 + 2$, if two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$. **(2)**

13. Find a quadratic polynomial whose zeros are 1 and -3. Verify the relation between the coefficients and zeros of the polynomial. **(2)**

14. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is -1 , then prove that the

product of other two zeros is b - a + 1. **(3)**

15. Find the zeroes of the quadratic polynomial $3x^2 - 2$ and verify the relationship between the zeroes and the coefficients. **(3)**

16. Find all the zeros of the polynomial $(2x^4 - 11x^3 + 7x^2 + 13x - 7)$, two of its zeros are $(3 + \sqrt{2})$ and $(3 - \sqrt{2})$. **(3)**

17. Find the zeroes of the quadratic polynomial $4y^2 - 15$ and verify the relationship between the zeroes and coefficient of polynomial. **(3)**

18. α, β, γ are the zeroes of the cubic polynomial $x^3 - 2x^2 + qx - r$. If $\alpha + \beta = 0$, then show that $2q = r$. **(4)**

19. α, β, γ are zeroes of cubic polynomial $x^3 - 12x^2 + 44x + c$. If $2\beta = \alpha + \gamma$, find the value of c. **(4)**

20. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$ if two value of its zeroes are $\sqrt{\frac{5}{3}}\ and\ -\sqrt{\frac{5}{3}}$. **(4)**

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Solution

1. c. $x^3 - 2x^2 - 7x + 14$

   Explanation: Let α, β, γ are the zeroes of the given polynomial. Given:
   $\alpha + \beta + \gamma = 2$ and $\alpha\beta + \beta\gamma + \gamma\alpha = -7$ and $\alpha\beta\gamma = -14$
   required polynomials = $\left[x^3 - (\alpha + \beta + \gamma) x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha) x - \alpha\beta\gamma\right|$
   = $\left[x^3 - 2x^2 + (-7) x - (-14)\right]$

   = $\left[x^3 - 2x^2 - 7x + 14\right]$
   required polynomial is $x^3 - 2x^2 - 7x + 14$

2. d. 3

   Explanation: A polynomial of degree 3 is called a cubic polynomial. A univariate cubic polynomial has the form $F(x) = a_3x^3 + a_2x^2 + a_1x + a_0$. An equation involving a cubic polynomial is called a cubic equation.

3. d. more than 3

   Explanation: The number polynomials having zeroes as –2 and 5 is more than 3. If 'S' is the sum and 'P' is the product of the zeroes then the corresponding family of quadratic polynomial is given by $p(x) = k\left(x^2 - \mathrm{S}x + \mathrm{P}\right)$ where k is any real number. Therefore putting different values of k, we can make more than 3 numbers of polynomials.

4. a. 0

   Explanation: A real number 'k' is said to be a zero of a polynomial p(x), if p(k) is equals to 0.

   Explanation: if P(x) is a Polynomial in x and k is any real number, then value of P(k) at x = k is denoted by P(k) is found by replacing x by k in P(x).

   e.g., In the polynomial $x^2–3x+2$,

   Replacing x by 1 gives,

   P(1) = 1–3+2 = 0

   Similarly, replacing x by 2 gives,

P(2) = 4−6+2 = 0

For a polynomial P(x), real number k is said to be zero of polynomial P(x), if P(k) = 0.

5. a. $\alpha + \beta = \alpha\beta$

Explanation: $\alpha+\beta=\frac{-b}{a}=\frac{-5}{1}$

And $\alpha\beta=\frac{c}{a}=\frac{-5}{1}$

$\therefore \alpha+\beta=\alpha\beta$

6. The number of zeroes is 2 as the graph intersects the x-axis at two points.

7. It is given that α and β are the zeros of the quadratic polynomial

$f(x)=ax^2+bx+c$

$\therefore \quad \alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$

Now, $\alpha^2+\beta^2$

$=\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta$

$=(\alpha+\beta)^2-2\alpha\beta$

$\therefore \quad \alpha^2+\beta^2=\left(\frac{-b}{a}\right)^2-\frac{2c}{a}=\frac{b^2-2ac}{a^2}$

8. We have,

f(x) = $4x^2$ - 4x - 3 = $4x^2$ - 6x + 2x - 3

= 2x(2x - 3) + 1(2x - 3) = (2x - 3) (2x + 1)

Now, f(x) = 0 $\Rightarrow$ (2x - 3)(2x + 1) = 0

$\therefore$ 2x - 3 = 0 or 2x + 1 = 0

or x = $\frac{3}{2}$, x = -$\frac{1}{2}$

Sum of zeros = $\frac{3}{2}$ + (-$\frac{1}{2}$) = $\frac{3-1}{2}$ = $\frac{2}{2}$ = 1

= $-\frac{(-4)}{4}$ = $\frac{4}{4}$ = 1= $-\frac{coeff.\,of\,x}{coeff.\,of\,x^2}$

Product of zeros = $\left(\frac{3}{2}\right)\times\left(-\frac{1}{2}\right)$ = -$\frac{3}{4}$

= $\frac{constant\ term}{coeff.\,of\,x^2}$

9. The given quadratic polynomial f(x) = ax^2 + bx + c

Here,a = a, b = b, c = c

$\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$

Now, $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=-\frac{b/a}{c/a}=-\frac{b}{c}$

10.
$$
\begin{array}{r}
5 \\
x-\sqrt{2}\,\overline{)\,5x-10} \\
5x-5\sqrt{2} \\
-\quad + \\
\hline
5\sqrt{2}-10
\end{array}
$$

Remainder = $5\sqrt{2}-10$

11. we have $x^2 - 4\sqrt{3}x + 3 = 0$

If α and β are the zeroes of $x^2 - 4\sqrt{3}x+3$

then, $\alpha+\beta=-\frac{b}{a}$

$\Rightarrow \alpha+\beta=-\frac{(-4\sqrt{3})}{1}$

$\alpha+\beta=4\sqrt{3}$

Now, $\alpha\beta=\frac{c}{a}$

$\Rightarrow \alpha\beta=\frac{3}{1}$

$\Rightarrow \alpha\beta=3$

$\therefore \alpha+\beta-\alpha\beta=4\sqrt{3}-3$

12. Here $f(x) = 2x^3 - 4x - x^2 + 2$

$=2x(x^2-2)-(x^2-2)$

$=(x^2-2)(2x-1)$

$=(x^2 - 2)(2x-1)=0$

$=(x - \sqrt{2})(x + \sqrt{2})(2x - 1)=0$

f(x)= if $(x - \sqrt{2})=0$ or $(x + \sqrt{2})=0$ or $(2x - 1)=0$

$\Rightarrow$ x = $\sqrt{2}$ or x = $-\sqrt{2}$ or x = = $\frac{1}{2}$

Hence the zeroes of f(x) are $\sqrt{2}$, $-\sqrt{2}$ and $\frac{1}{2}$

13. Let $\alpha=1$ and $\beta=-3$

Sum of zeros = $(\alpha+\beta) = 1 + (-3) = -2$............... (1)

Product of zeros = $\alpha\beta = 1 \times (-3) = -3$ .... (2)

So, the required polynomial is

$x^2 - (\alpha+\beta)x + \alpha\beta = x^2 - (-2)x + (-3)$

= $x^2 + 2x - 3$

So here the coefficients are

a=1, b=2 and c=-3

so $-\frac{b}{a} = -\frac{2}{1} = -2.......(3)$

$\frac{c}{a} = \frac{-3}{1} = -3.......(4)$

Hence from (1) & (3) and from (2) & (4)

$\alpha + \beta = -\frac{b}{a}$

and $\{\alpha\beta\} = \frac{c}{a}$

14. Let $p(x) = x^3 + ax^2 + bx + c$

As -1 is one of the zeroes of p(x), p(-1) = 0

$\Rightarrow (-1)^3 + a(-1)^2 + b(-1) + c = 0$

$\Rightarrow -1 + a - b + c = 0$

$\Rightarrow c = b - a + 1$ .... (i)

Let α, β be two other zeroes of p(x),

then product of zeroes =-1× α × β =$-\frac{Constant\ term}{coefficient\ of\ x^3}$

$\Rightarrow (-1)(\alpha\beta) = -\frac{c}{1}$

$\Rightarrow -\alpha\beta = -c$

$\Rightarrow \alpha\beta = c$

$\Rightarrow \alpha\beta = b - a + 1$ [using (i)]

Hence, the product of other two zeroes of the given cubic polynomial is b – a + 1.

15. Here, $p(x) = 3x^2 - 2$.

Now p(x) = 0

$\Rightarrow \quad 3x^2 - 2 = 0$

$\Rightarrow 3x^2 = 2$

$\Rightarrow \quad x^2 = \frac{2}{3}$

$\Rightarrow x = \pm\sqrt{\frac{2}{3}}$

Therefore, zeroes are $\sqrt{\frac{2}{3}}$ and $-\sqrt{\frac{2}{3}}$.

If $p(x) = 3x^2 - 2$, then a = 3, b = 0 and c = -2

Now, sum of zeroes = $\sqrt{\frac{2}{3}} + \left(-\sqrt{\frac{2}{3}}\right) = 0$ ..... (i)

Also, $\frac{-b}{a} = \frac{-0}{3} = 0$ .... (ii)

From (i) and (ii)

Sum of zeroes= $\frac{-b}{a}$

and product of zeroes = $\sqrt{\frac{2}{3}} \times -\sqrt{\frac{2}{3}} = \frac{-2}{3}$ .... (iii)

Also, $\frac{c}{a} = \frac{-2}{3}$ .... (iv)

From (iii) and (iv)

Product of zeroes = $\frac{c}{a}$

16. we are given that the two zeroes of given polynomial are $(3 + \sqrt{2})$ and $(3 - \sqrt{2})$.

The given quadratic polynomial is

$p(x) = 2x^4 - 11x^3 + 7x^2 + 13x - 7$

Sum of $(3 + \sqrt{2})$ and $(3 - \sqrt{2}) = (3 + \sqrt{2}) + (3 - \sqrt{2}) = 6$

Product of $(3 + \sqrt{2})$ and $(3 - \sqrt{2}) = (3 + \sqrt{2})(3 - \sqrt{2}) = 9 - 2 = 7$

Polynomial whose zeros are $3 + \sqrt{2}$ and $3 - \sqrt{2}$ is

x^2 - (sum of zeros)x + (Product of zeros) = $x^2 - 6x + 7$

Now we Divide p(x) by $x^2 - 6x + 7$ as:

$$
\begin{array}{r|l}
 & 2x^2 + x - 1 \\
x^2 - 6x + 7 & 2x^4 - 11x^3 + 7x^2 + 13x - 7 \\
 & 2x^4 - 12x^3 + 14x^2 \\
 & - \quad + \quad - \\
\hline
 & x^3 - 7x^2 + 13x \\
 & x^3 - 6x^2 + 7x \\
 & - \quad + \quad - \\
\hline
 & -x^2 + 6x - 7 \\
 & -x^2 + 6x - 7 \\
 & + \quad - \quad + \\
\hline
 & 0
\end{array}
$$

Therefore, Quotient = $2x^2 + x - 1$ and remainder = 0.

Other two zeros of polynomial p(x) are also the zeros of q(x) i.e., $q(x) = 2x^2 + x - 1 = 2x^2 + 2x - x - 1$ (by splitting the midddle term)

$= 2x(x + 1) - (x + 1) = (x + 1)(2x - 1)$

In order to find the values of x, put(x) = 0

$\Rightarrow (x + 1)(2x - 1) = 0$

$\Rightarrow$ Either $x + 1 = 0$ or $2x - 1 = 0$

$\Rightarrow$ Either $x = -1$ or $x = \frac{1}{2}$

$\therefore$ The zeros of given polynomial p(x) are

$\frac{1}{2}$, -1, $(3 + \sqrt{2})$ and $(3 - \sqrt{2})$

17. We have to find the zeroes of the quadratic polynomial $4y^2 - 15$ and verify the relationship between the zeroes and coefficient of polynomial.

Let $f(y) = 4y^2 - 15$

Compare it with the quadratic $ay^2 + by + c$.

Here, coefficient of $y^2 = 4$, coefficient of y = 0 and constant term = - 15.

Now $4y^2 - 15 = (2y)^2 - (\sqrt{15})^2$

$= (2y + \sqrt{15})(2y - \sqrt{15})$

The zeroes of f(y) are given by $f(y) = 0$

$\Rightarrow (2y) \quad \sqrt{15})(2y - \sqrt{15}) + \quad = 0$

$\Rightarrow (2y) + \sqrt{15}) = 0$ or $(2y - \sqrt{15}) = 0$

$\Rightarrow 2y = -\sqrt{15}$ or $2y = \sqrt{15}$

$\Rightarrow y = -\frac{\sqrt{15}}{2}$ or $y = \frac{\sqrt{15}}{2}$

Hence, the zeroes of the given quadratic polynomial are $-\frac{\sqrt{15}}{2}, \frac{\sqrt{15}}{2}$

Verification of relationship between zeroes and coefficients

Sum of the zeroes = $-\frac{\sqrt{15}}{2} + \frac{\sqrt{15}}{2} = \frac{-\sqrt{15} + \sqrt{15}}{2} = \frac{0}{2} = 0 = \frac{0}{4}$
$= \frac{coefficient\ of\ y}{coefficient\ of\ y^2}$

Product of zeroes = $-\frac{\sqrt{15}}{2} \times \frac{\sqrt{15}}{2} = -\frac{15}{4} = \frac{constant\ term}{coefficient\ of\ y^2}$.

18. $p(x) = x^3 - 2x^2 + qx - r$.

Here, a = 1, b = -2, c = q, d = -r

Sum of zeroes = $\frac{-b}{a} \Rightarrow \alpha + \beta + \gamma = \frac{-(-2)}{1} = 2$

$\Rightarrow 0 + \gamma = 2$

$\Rightarrow \gamma = 2$............(i)

Also, $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} \Rightarrow \alpha\beta + \gamma(\alpha + \beta) = \frac{q}{1}$

$\Rightarrow \alpha\beta + \gamma \times 0 = q \Rightarrow \alpha\beta = q$ .... (ii)

and $\alpha . \beta \cdot \gamma = \frac{-d}{a}$

$\Rightarrow$ q.2 = -(-r) (Using (i) and (ii))

$\Rightarrow$ 2q = r

19. Given, $p(x) = x^3 - 12x^2 + 44x + c$

$\alpha + \beta + \gamma = \frac{-(-12)}{1} = 12$

Also, $\alpha + \gamma = 2\beta$ (given)

$\Rightarrow \quad 2\beta + \beta = 12 \Rightarrow \beta = 4$

Now, $\alpha \cdot \beta \cdot \gamma = -c$

$\Rightarrow \quad \alpha \cdot \gamma . 4 = -c$

$\Rightarrow \alpha \cdot \gamma = -\frac{c}{4}$

Also, $\alpha\beta + \beta\gamma + \alpha\gamma = 44$

$\Rightarrow \beta(\alpha + \gamma) + \left(-\frac{c}{4}\right) = 44$

$\Rightarrow \beta \times 2\beta - \frac{c}{4} = 44$

$\Rightarrow 4 \times 2 \times 4 - \frac{c}{4} = 44$

$\Rightarrow -\frac{c}{4} = 44 - 32$

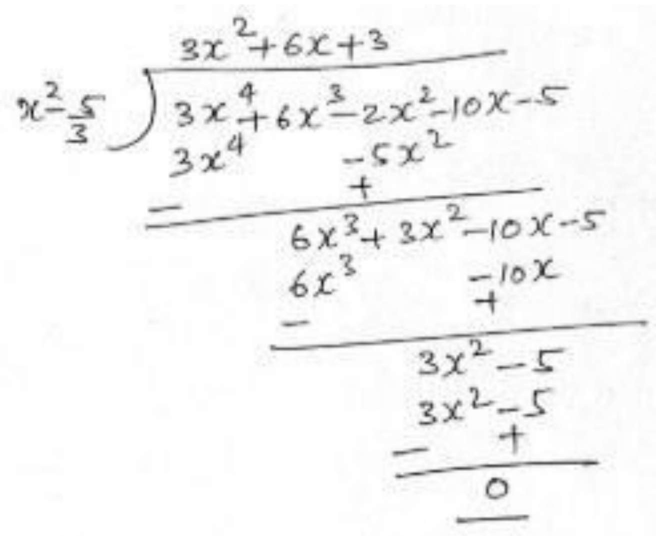
$\Rightarrow -\frac{c}{4} = 12$

$\Rightarrow c = -48$

20. Two zeroes of $p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$ are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

It means $\left(x - \sqrt{\frac{5}{3}}\right)\left(x \quad \sqrt{\frac{5}{3}}\right) = \left(x^2 - \frac{5}{3}\right) +$ is a factor of p(x)

Applying division algorithm to find more factors we get,



Factorising the quotient to get other factors of given polynomial.

$q(x) = 3x^2 + 6x + 3$

$= 3(x^2 + 2x + 1)$

$= 3(x + 1)^2$

Now to find other zeroes,

$q(x) = 0$

$3(x + 1)^2 = 0$

$(x + 1) = 0$ or $(x + 1) = 0$

$x = -1$ or $x = -1$

Thus, the zeroes of the given polynomial are $\pm\sqrt{\frac{5}{3}}, -1, -1$