

CBSE Test Paper 05
CH-2 Polynomials

1. Which of the following expression is a monomial
 - a. $4x^3$
 - b. $x^6 + 2x^2 + 2$
 - c. None of these
 - d. $3 + x$

2. The value of $(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$ is
 - a. $3(a + b)(b + c)(c + a)(a - b)(b - c)(c - a)$
 - b. $3(a - b)(b - c)(c - a)$
 - c. $3(a + b)(b + c)(c + a)$
 - d. none of these

3. The value of $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})(x + y)(x^2 + y^2)$ is
 - a. $(x^4 + y^4)$
 - b. $(x^4 - y^4)$
 - c. $(x + y)^4$
 - d. $(x - y)^4$

4. A polynomial containing one nonzero term is called a_____.
 - a. trinomial
 - b. binomial
 - c. none of these

d. monomial

5. The value of $(x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x-a)(x-b)(x-c)$ when $a + b + c = 3x$, is

a. 1

b. 2

c. 3

d. 0

6. Fill in the blanks:

A polynomial containing three non-zero terms is called a _____.

7. Fill in the blanks:

The coefficient of x in the expansion of $(x + 3)^3$ is _____.

8. Write the degree of the following polynomial: $5t - \sqrt{7}$

9. Whether the following are zero of the polynomial, indicated against them. $p(x) = lx + m$, $x = -\frac{m}{l}$

10. Expand: $\left(\frac{1}{x} + \frac{y}{3}\right)^3$

11. Find the following product: $(2x - y + 3z)(4x^2 + y^2 + 9z^2 + 2xy + 3yz - 6xz)$

12. Determine the remainder when the polynomial $p(x) = x^4 - 3x^2 + 2x + 1$ is divided by $x - 1$.

13. Simplify: $\frac{(a^2-b^2)^3 + (b^2-c^2)^3 + (c^2-a^2)^3}{(a-b)^3 + (b-c)^3 + (c-a)^3}$

14. Factorize: $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

15. If $x - 3$ and $x - \frac{1}{3}$ are both factors of $px^2 + 5x + r$, then show that $p = r$

CBSE Test Paper 05
CH-2 Polynomials

Solution

1. (a) $4x^3$

Explanation: $4x^3$ because monomial means only one term in an expression.

2. (a) $3(a + b)(b + c)(c + a)(a - b)(b - c)(c - a)$

Explanation:

$$(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$$

Here,

$$a^2 - b^2 + b^2 - c^2 + c^2 - a^2 = 0$$

Therefore,

$$(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3 = 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2) \quad [\text{Since } x^3 + y^3 + z^3 = 3xyz \text{ if } x + y + z = 0]$$

$$(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3 = 3(a + b)(b + c)(c + a)(a - b)(b - c)(c - a)$$

3. (b) $(x^4 - y^4)$

Explanation:

$$\begin{aligned} & (\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})(x + y)(x^2 + y^2) \\ &= [(\sqrt{x})^2 - (\sqrt{y})^2](x + y)(x^2 + y^2) \\ &= (x - y)(x + y)(x^2 + y^2) \\ &= [(x)^2 - (y)^2](x^2 + y^2) \\ &= (x^2 - y^2)(x^2 + y^2) \\ &= (x^2)^2 - (y^2)^2 \end{aligned}$$

$$= x^4 - y^4$$

4. (d) monomial

Explanation: A polynomial containing one nonzero term is called a monomial.

Example: $3x, 5x^2, y^3$

5. (d) 0

Explanation:

$$(x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)(x - b)(x - c)$$

$$= [x - a + x - b + x - c]$$

$$[x - a^2) + (x - b^2) + (x - c^2) - (x - a)(x - b) - (x - b)(x - c) - (x - c)(x -$$

$$= [3x - (a + b + c]$$

$$[(x - a)^2 + (x - b)^2 + (x - c)^2 - (x - a)(x - b) - (x - b)(x - c) - (x - c)(x -$$

$$= 3x - 3x$$

$$[(x - a)^2 + (x - b)^2 + (x - c)^2 - (x - a)(x - b) - (x - b)(x - c) - (x - c)(x -$$

$$=$$

$$[0] [(x - a)^2 + (x - b)^2 + (x - c)^2 - (x - a)(x - b) - (x - b)(x - c) - (x - c)(x -$$

$$= 0$$

6. trinomial

7. 27

8. Term with the highest power of $t = 5t$

Exponent of t in this term = 1

\therefore Degree of this polynomial = 1

9. $p(-\frac{m}{l}) = l(-\frac{m}{l}) + m = -m + m = 0$

$\therefore -\frac{m}{l}$ is a zero of $p(x)$.

10. $(x+y)^3 = x^3 + y^3 + 3x^2y + 3xy^2$

$$\begin{aligned} \left[\frac{1}{x} + \frac{y}{3} \right]^3 &= \left(\frac{1}{x} \right)^3 + 3 \left(\frac{1}{x} \right)^2 \frac{y}{3} + 3 \frac{1}{x} \left(\frac{y}{3} \right)^2 + \left(\frac{y}{3} \right)^3 \\ &= \left(\frac{1}{x} \right)^3 + 3 \cdot \left(\frac{1^2}{x^2} \right) \frac{y}{3} + 3 \cdot \frac{1}{x} \frac{y^2}{3^2} + \frac{y^3}{3^3} \\ &= \frac{1}{x^3} + \frac{y}{x^2} + \frac{y^2}{3x} + \frac{y^3}{27} \end{aligned}$$

$$\begin{aligned} 11. (2x - y + 3z) (4x^2 + y^2 + 9z^2 + 2xy + 3yz - 6xz) \\ &= (2x + (-y) + 3z) \{ (2x)^2 + (-y)^2 + (3z)^2 - 2x \times (-y) - (-y) \times (3z) - 2x \times 3z \} \\ &= (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca), \text{ where } a = 2x, b = -y, c = 3z \\ &= a^3 + b^3 + c^3 - 3abc \\ &= (2x)^3 + (-y)^3 + (3z)^3 - 3 \times 2x \times (-y) \times 3z \\ &= 8x^3 - y^3 + 27z^3 + 18xyz \end{aligned}$$

12. By remainder theorem, the required remainder is equal to $p(1)$.

$$\text{Now, } p(x) = x^4 - 3x^2 + 2x + 1$$

$$\Rightarrow p(1) = (1)^4 - 3 \times 1^2 + 2 \times 1 + 1 = 1 - 3 + 2 + 1 = 1$$

$$\text{Hence, required remainder} = p(1) = 1$$

13. We have,

$$(a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2) = 0$$

$$\therefore (a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3 = 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$$

$$\Rightarrow (a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3 = 3(a - b)(a + b)(b - c)(b + c)(c - a)(c + a)$$

Similarly, we have,

$$(a - b) + (b - c) + (c - a) = 0$$

$$\Rightarrow (a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a)$$

$$\begin{aligned} \therefore \frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3} \\ = \frac{3(a - b)(a + b)(b - c)(b + c)(c - a)(c + a)}{3(a - b)(b - c)(c - a)} = (a + b)(b + c)(c + a) \end{aligned}$$

$$14. 4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$$

The given expression can be re written as

$$(2x)^2 + (3y)^2 + (-4z)^2 + 2 \times 2x \times 3y + 2 \times 3y \times (-4z) + 2 \times (-4z) \times 2x$$

As we know, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$(2x)^2 + (3y)^2 + (-4z)^2 + 2 \times 2x \times 3y + 2 \times 3y \times (-4z) + 2 \times (-4z) \times 2x,$$

$$\Rightarrow (2x + 3y - 4z)^2 = (2x + 3y - 4z)(2x + 3y - 4z)$$

$$\Rightarrow 4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz = (2x + 3y - 4z)(2x + 3y - 4z)$$

15. $\therefore x - 3$ and $x - \frac{1}{3}$ are factors of

$$px^2 + 5x + r \therefore x = 3, x = \frac{1}{3}$$

zero of $px^2 + 5x + r$

Putting $x = 3$ in given polynomial,

$$\therefore p(3)^2 + 5 \times 3 + r = 0$$

$$9p + 15 + r = 0$$

$$9p + r = -15 \text{ ----- (1)}$$

Again putting $x = \frac{1}{3}$ in given polynomial,

$$p\left(\frac{1}{3}\right)^2 + 5 \times \frac{1}{3} + r = 0$$

$$\frac{p}{9} + \frac{5}{3} + r = 0$$

$$\frac{p+15+9r}{9} = 0$$

$$p + 9r = -15 \text{ ----- (2)}$$

From eq.(1) and eq.(2), we have,

$$9p + r = p + 9r$$

$$9p - p = 9r - r$$

$$8p = 8r$$

$$p = r$$

Hence proved