CBSE Test Paper 05

Chapter 2 Polynomials

1. If one zero of the polynomial $p(x) = (a^2 + 9)x^2 + 45x + 6a$ is reciprocal of the other, then the value of 'a' is **(1)**
   a. 2
   b. 3
   c. 0
   d. 1
2. If $x - 2$ is a factor of the polynomial $3x^3 - 7x^2 + kx - 16$, then the value of 'k' is **(1)**
   a. -10
   b. 10
   c. -2
   d. 2
3. The zeroes of a polynomial $x^2 - 7x + 12$ are **(1)**
   a. both positive
   b. both negative
   c. both equal
   d. one positive and one negative
4. Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, then the product of the other two zeroes is **(1)**
   a. $\frac{-c}{a}$
   b. $\frac{b}{a}$
   c. $\frac{-b}{a}$
   d. $\frac{c}{a}$
5. If 'α' and 'β' are the zeroes of the polynomial $3x^2 + 11x - 4$, then the value of $\alpha^2 + \beta^2$ is **(1)**
   a. $\frac{150}{9}$
   b. $\frac{145}{9}$
   c. $\frac{152}{9}$
   d. $\frac{144}{9}$

6. If α, β are zeroes of $x^2 + 5x + 5$, find the value of $\alpha^{-1} + \beta^{-1}$. **(1)**
7. If -4 is a zero of the polynomial $x^2 - x - (2k + 2)$ then find the value of k. **(1)**
8. Find a cubic polynomial with the sum, sum of the product of its zeros taken two at a time, and product of its zeroes as 3, -1 and - 3 respectively. **(1)**
9. If $p(x) = ax^2 + bx + c$. If a + c = b, then find one of its zeroes. **(1)**
10. If α and β are the roots of equation $ax^2 - bx + c = 0$, then find the value of $\alpha + \beta$. **(1)**
11. If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, then what will be the quotient and remainder? **(2)**
12. Find the zeroes of $4x^2 + 24x + 36$ and verify the relationship between the zeroes and their coefficients. **(2)**
13. Divide the polynomial $p(x) = x^2 - 5x + 16$ by the polynomial $g(x) = x - 2$ and find the quotient and the remainder. **(2)**
14. On dividing $x^3 + 4x^2 + 3x + 2$ by g(x), quotient and reminder were $(x^2 – 2)$ and $(5x + 10)$ respectively. Find g(x). **(3)**
15. Find a cubic polynomial whose zeros are 3, 5 and -2. **(3)**
16. Find the quadratic polynomial whose zeroes are 2and -6 respectively. Verify the relation between the coefficients and zeroes of the polynomial. **(3)**
17. Obtain all the zeroes of $2x^4 - 7x^3 - 13x^2 + 63x - 45$ if two of its zeroes are 1 and 3. **(3)**
18. If α and β are the zeroes of the polynomial $x^2 + 4x + 3$, find the polynomial whose zeroes are $1 + \frac{\beta}{\alpha}$ and $1 + \frac{\alpha}{\beta}$.. **(4)**
19. Polynomial $x^4 + 7x^3 + 7x^2 + px + q$ is exactly divisible by $x^2 + 7x + 12$, then find the value of p and q. **(4)**
20. If two zeroes of a polynomial $x^3 + 5x^2 + 7x + 3$ are - 1 and - 3, then find the third zero. **(4)**

CBSE Test Paper 05

Chapter 2 Polynomials

Solution

1. b. 3

   Explanation: Let one zero be β then the other zero will be $\frac{1}{\alpha}$

   Since $\alpha\beta = \frac{c}{a} \Rightarrow \alpha \times \frac{1}{\alpha} = \frac{6a}{a^2+9}$

   $\Rightarrow 1 = \frac{6a}{a^2+9}$

   $\Rightarrow 6a = a^2 + 9$

   $\Rightarrow a^2 - 6a + 9 = 0$

   $\Rightarrow (a-3)(a-3) = 0$ $a - 3 = 0$ and $a - 3 = 0$

   $\Rightarrow a = 3$ and $a = 3$

2. b. 10

   Explanation: If the polynomial $3x^3 - 7x^2 + kx - 16$ is exactly divisible by $x - 2$, then

   $p(2) = 0$

   $\Rightarrow 3(2)^3 - 7(2)^2 + k \times 2 - 16 = 0$

   $\Rightarrow 24 - 28 + 2k - 16 = 0$

   $\Rightarrow -20 + 2k = 0$

   $\Rightarrow k = 10$

3. a. both positive

   Explanation: x^2 -$7x + 12$

   = $x^2 - 4x - 3x + 12$=0

   = $x(x-4) - 3(x-4)$=0

   = $(x-4)(x-3)$=0

   $\therefore x - 4 = 0$ or $x - 3 = 0$

   $\Rightarrow x = 4$ or $x = 3$

4. d. $\frac{c}{a}$

   Explanation: Let α, β, γ are the zeroes of the given polynomial. Given : $\alpha = 0$

   To find: $\beta\gamma$ Since, $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

   $\therefore 0 \times \beta + \beta\gamma + \gamma \times 0 = \frac{c}{a} \Rightarrow \beta\gamma = \frac{c}{a}$

5. b. $\frac{145}{9}$

Explanation: Here $a = 3, b = 11, c = -4$

Since $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

= $\left(\frac{-b}{a}\right)^2 - 2 \times \frac{c}{a}$ = $\frac{b^2}{a^2} - \frac{2c}{a}$ = $\frac{b^2 - 2ac}{a^2}$

Putting the values of a, b and c, we get = $\frac{(11)^2 - 2 \times 3 \times (-4)}{(3)^2}$

= $\frac{121+24}{9}$

= $\frac{145}{9}$

6. We know that sum of roots = $\alpha + \beta = -\frac{b}{a}$

$\Rightarrow \quad \alpha + \beta = -5$

and product of roots = $\alpha\beta = \frac{c}{a} \Rightarrow \alpha\beta = 5$ Now the given expression is :

$\alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta}$ = $\frac{\alpha+\beta}{\alpha\beta} = \frac{-5}{5}$ = -1

7. Given that, -4 is a zero of the polynomial f(x) = $x^2 - x - (2k + 2)$, so, we have

f(-4) = 0

$\Rightarrow (-4)^2 - (-4) - 2k - 2 = 0$

$\Rightarrow 16 + 4 - 2k - 2 = 0$

$\Rightarrow 18 - 2k = 0$

$\Rightarrow 2k = 18$

$\Rightarrow k = 9.$

8. Any cubic polynomial is of the form $ax^3 + bx^2 + cx + d$

= x^3 - (sum of the zeroes) x^2 + (sum of the products of its zeroes taken two at a time) x - (product of the zeroes)

= $x^3 - 3x^2 + (-1)x + (-3)$

= $x^3 - 3x^2 - x - 3$

Hence, required cubic polynomial is $x^3 - 3x^2 - x - 3$

9. We have function p(x) = $ax^2 + bx + c$ and a + c = b

using remainder theorem by putting x = -1 we get

p (-1) = $a(-1)^2 + b(-1) + c$

= a - b + c = a + c - b

= b - b = 0

∴ One zero is -1.

10. Sum of the roots = $-\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

or, $\alpha+\beta=-\left(\frac{-b}{a}\right)=\frac{b}{a}$

11. On long division of $6x^4 + 8x^3 + 17x^2 + 21x + 7$ by $3x^2 + 4x + 1$ we get

$$\begin{array}{r|l} & 2x^2 + 5 \\ 3x^2 + 4x + 1 & 6x^4 + 8x^3 + 17x^2 + 21x + 7 \\ & \underline{-6x^4 \mp 8x^3 \mp 2x^2} \\ & \quad\quad 15x^2 + 21x + 7 \\ & \quad\quad \underline{-15x^2 \mp 20x \mp 5} \\ & \quad\quad\quad\quad x + 2 \end{array}$$

Quotient = $2x^2 + 5$, remainder = x + 2

12. p(x) = $4x^2 + 24x + 36$

For zeroes, p(x) = 0

$\Rightarrow$ $4x^2 + 24x + 36 = 0$

$\Rightarrow$ $4(x^2 + 6x + 9) = 0$

$\Rightarrow$ $(x^2 + 3x + 3x + 9) = 0$

$\Rightarrow$ (x + 3) (x + 3) = 0

$\Rightarrow$ x + 3 = 0 or x + 3 = 0

$\Rightarrow$ x = -3, x = -3

∴ Zeroes are -3, -3.

After comparing $4x^2 + 24x + 36$ with $ax^2 + bx + c$, we get

Now, a = 4, b = 24, c = 36

$\frac{-b}{a}=\frac{-24}{4}$ = -6 ....(i)

Sum of zeroes = -3 + (-3) = -6 ....(ii)

From (i) and (ii)

Sum of zeroes $=-\frac{b}{a}$

Also, $\frac{c}{a}=\frac{36}{4}=9$ .... (iii) and, Product of zeroes = (-3) $\times$ (-3) = 9 .... (iv)

From (iii) and (iv)

Product of zeroes $= \frac{c}{a}$

13.

$$\begin{array}{r|l} & x-3 \\ \hline x-2 & x^2-5x+16 \\ & x^2-2x \\ & -\quad + \\ \hline & -3x+16 \\ & -3x+6 \\ & +\quad - \\ \hline & 10 \\ \hline \end{array}$$

$\Rightarrow$ Quotient = x - 3, Remainder = 10

14. Using, Dividend = Divisor $\times$ Quotient + Remainder

$x^3 + 4x^2 + 3x + 2 = g(x) \times (x^2 - 2) + (5x + 10)$

$\Rightarrow (x^3 + 4x^2 + 3x + 2) - (5x + 10) = (x^2 - 2) \times g(x)$

$\Rightarrow x^3 + 4x^2 + 3x + 2 - 5x - 10 = (x^2 - 2) \times g(x)$

$\Rightarrow x^3 + 4x^2 - 2x - 8 = (x^2 - 2) \times g(x)$ ....(i)

$\Rightarrow (x^2 - 2)$ is a factor of $x^3 + 4x^2 - 2x - 8$

$$\begin{array}{r|l} & x+4 \\ \hline x^2-2 & x^3+4x^2-2x-8 \\ & x^3 \qquad -2x \\ & - \qquad\quad + \\ \hline & 4x^2 \qquad -8 \\ & 4x^2 \qquad -8 \\ & - \qquad\quad + \\ \hline & 0 \\ \hline \end{array}$$

$\Rightarrow x^3 + 4x^2 - 2x - 8 = (x^2 - 2)(x + 4)$

$\therefore$ g(x) = (x + 4) [On comparing with (i)]

15. Let α, β and γ be the zeroes of the given polynomial.

Then, we have α = 3, β = 5 and γ = -2

Hence

$\alpha + \beta + \gamma$ = 3 + 5 - 2 = 6 ....(1)

$\alpha\beta + \beta\gamma + \gamma\alpha$ = 3(5) + 5(-2) + (-2)3 = 15 - 10 - 6 = -1 ....(2)

$\alpha\beta\gamma$ = 3(5)(-2) = -30 ....(3)

Now, a cubic polynomial whose zeros are α, β and γ is equal to p(x) = x^3 - $(\alpha+\beta+\gamma)x^2+(\alpha\beta+\beta y+\gamma\alpha)x-\alpha\beta\gamma$ On substituting values from (1),(2) and (3) we get

$$p(x)=x^3-(6)x^2+(-1)x-(-30)$$

= x^3 - $6x^2$ - x + 30

16. Let $\alpha=2$ and $\beta=-6$.
Then, required polynomial is given by $x^2-(\alpha+\beta)x+\alpha\beta=x^2+4x-12$

Also, here sum of zeroes = $\alpha+\beta$= $2+(-6)=-4$ = $\frac{-(Coefficients(x))}{coefficient(x^2)}$
and product of zeroes=$\alpha\beta$= $2(-6)=-12$ = $\frac{Constant\_term}{Coefficient(x^2)}$

Polynomial = x^2 - (sum of zeros)x + product of zeros
Polynomial = $x^2-(-4)x+(-12)$= $x^2+4x-12$
Hence, the required polynomial is $x^2+4x-12$ and the relationship between zeroes and coefficients is verified.

17. Since, two zeroes are 1 and 3, therefore $(x-1)(x-3)=x^2-4x+3$ is a factor of the given polynomial.

Now, we apply the division algorithm to the given polynomial and x^2 - 4x + 3

$$
\begin{array}{r|l}
 & 2x^2+x-15 \\
x^2-4x+3 & 2x^4-7x^3-13x^2+63x-45 \\
 & 2x^4-8x^3+6x^2 \\
 & -\quad+\quad- \\
\hline
 & x^3-19x^2+63x-45 \\
 & x^3-4x^2+3x \\
 & -\quad+\quad- \\
\hline
 & -15x^2+60x-45 \\
 & -15x^2+60x-45 \\
 & +\quad-\quad+ \\
\hline
 & 0
\end{array}
$$

So, $2x^4-7x^3-13x^2+63x-45$
$=\left(x^2-4x+3\right)\left(2x^2+x-15\right)$
Now, $2x^2+x-15=2x^2+6x-5x-15$
|by splitting the middle term
$=2x(x+3)-5(x+3)=(x+3)(2x-5)$

So, its zeroes are -3 and $\frac{5}{2}$

Therefore, all the zeroes of the given fourth-degree polynomial are 1, 3, -3 and $\frac{5}{2}$.

18. Since α and β are the zeroes of the quadratic polynomial $x^2 + 4x + 3$

So, $\alpha + \beta = -4$

and $\alpha\beta = 3$

Sum of zeroes of new polynomial $= 1 + \frac{\beta}{\alpha} + 1 + \frac{\alpha}{\beta}$

$= \frac{\alpha\beta + \beta^2 + \alpha\beta + \alpha^2}{\alpha\beta}$

$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(-4)^2}{3} = \frac{16}{3}$

Product of zeroes = $\left(1 + \frac{\beta}{\alpha}\right)\left(1 + \frac{\alpha}{\beta}\right)$

$= \left(\frac{\alpha+\beta}{\alpha}\right)\left(\frac{\beta+\alpha}{\beta}\right)$

$= \frac{(\alpha+\beta)^2}{\alpha\beta}$

$= \frac{(-4)^2}{3} = \frac{16}{3}$

So required polynomial = x^2 - (Sum of the zeroes)x + Product of the zeroes

$= x^2 - \left(\frac{16}{3}\right)x + \frac{16}{3}$

$= \left(x^2 - \frac{16}{3}x + \frac{16}{3}\right)$

$= \frac{1}{3}\left(3x^2 - 16x + 16\right)$

19. Factors of $x^2 + 7x + 12$:

$x^2 + 7x + 12 = 0$

$\Rightarrow x^2 + 4x + 3x + 12 = 0$

$\Rightarrow x(x + 4) + 3(x + 4) = 0$

$\Rightarrow (x + 4)(x + 3) = 0$

$\Rightarrow x = -4, -3$ ...(i)

Since $p(x) = x^4 + 7x^3 + 7x^2 + px + q$

If p(x) is exactly divisible by $x^2 + 7x + 12$, then x = - 4 and x = - 3 are its zeroes. So putting x = - 4 and x = - 3.

$p(-4) = (-4)^4 + 7(-4)^3 + 7(-4)^2 + p(-4) + q$

but p(- 4) = 0

$\therefore$ 0 = 256 - 448 + 112 - 4p + q

0 = - 4p + q - 80

$\Rightarrow$ 4p - q = - 80 ...(i)

and p(-3) = $(-3)^4 + 7(-3)^3 + 7(-3)^2$ + p(-3) + q

but p(-3) = 0

$\Rightarrow$ 0 = 81-189 + 63 - 3p + q

$\Rightarrow$ 0 = -3p + q -45

$\Rightarrow$ 3p -q = -45........... (ii)

$$\begin{array}{r} 4p - q = -80 \\ 3p - q = -45 \\ - \quad + \quad\quad + \\ \hline p = -35 \end{array}$$

On putting the value of p in eq. (i),we get,

$4(-35) - q = -80$

$\Rightarrow -140 - q = -80$

$\Rightarrow \quad -q = 140 - 80$

$\Rightarrow \quad -q = 60$

$\therefore q = -60$

Hence, p = -35, q = -60

20. x = -1 and x = -3 are zeroes.

$\Rightarrow$ (x + 1)(x + 3) = x^2 + 4x + 3

$$\begin{array}{r|l} x^2 + 4x + 3 & x^3 + 5x^2 + 7x + 3 \;(x + 1 \\ & x^3 + 4x^2 + 3x \\ & - \quad - \\ \hline & x^2 + 4x + 3 \\ & x^2 + 4x + 3 \\ & - \quad - \\ \hline & 0 \end{array}$$

Since remainder = 0, therefore (x + 1) is factor of $x^3 + 5x^2 + 7x + 3$.

So, required zero is given by putting x + 1 = 0

$\Rightarrow$ x = -1

$\therefore$ The third zero is -1.