

CBSE Test Paper 04
CH-8 Quadrilaterals

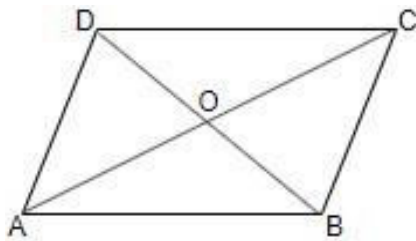
1. In Parallelogram ABCD, bisectors of angles A and B intersect each other at O. The measure of $\angle AOB$ is

- a. 90°
- b. 30°
- c. 60°
- d. 120°

2. A quadrilateral ABCD is a parallelogram if

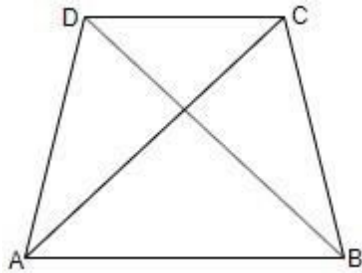
- a. $\angle A = 60^\circ, \angle C = 60^\circ, \angle B = 120^\circ$
- b. $AB = CD$
- c. $AB = AD$
- d. $AB \parallel BC$

3. In the given figure, ABCD is a Rhombus. Then,



- a. $(AC^2 + BD^2) = 3AB^2$
- b. $AC^2 + BD^2 = 4AB^2$
- c. $AC^2 + BD^2 = AB^2$
- d. $AC^2 + BD^2 = 2AB^2$

4. In a Trapezium ABCD, if $AB \parallel CD$, then $(AC^2 + BD^2) = ?$



- a. $BC^2 + AD^2 + 2BC \cdot AD$
- b. $AB^2 + CD^2 + 2AB \cdot CD$
- c. $AB^2 + CD^2 + 2AD \cdot BC$
- d. $BC^2 + AD^2 + 2AB \cdot CD$
5. The angle between two altitudes of a Parallelogram through the vertex of an obtuse angle of the Parallelogram is 60° . Find the angles of the Parallelogram
- a. $200^\circ, 100^\circ, 30^\circ, 30^\circ$
- b. $110^\circ, 50^\circ, 105^\circ, 105^\circ$
- c. $150^\circ, 150^\circ, 30^\circ, 30^\circ$
- d. $120^\circ, 60^\circ, 120^\circ, 60^\circ$

6. Fill in the blanks:

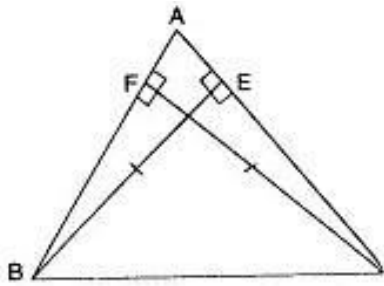
A diagonal of a parallelogram, divides it into two _____ triangles.

7. Fill in the blanks:

The ratio of the angles P, Q, R and S of a quadrilateral PQRS is $3 : 6 : 4 : 5$, then the measure of each angle of the quadrilateral is _____.

8. Three angles of a quadrilateral ABCD are equal. Is it a parallelogram?
9. Three angles of a quadrilateral are respectively equal to $110^\circ, 50^\circ$ and 40° . Find its fourth angle.

10. Prove that $\triangle ABC$ is an isosceles triangle, if altitude $BE =$ altitude CF .



11. Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.
12. An exterior angle of a triangle is 110° , and one of the interior opposite angles is 30° . Find the other two angles of the triangle.
13. A diagonal of a parallelogram bisects one of its angles. Show that it is a rhombus.
14. In the following figure, D, E and F are respectively the mid-points of sides BC, CA and AB of an equilateral triangle $\triangle ABC$. Prove that $\triangle DEF$ is also an equilateral triangle.
15. P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD in which $AC = BD$. Prove that PQRS is a rhombus.

CBSE Test Paper 04
CH-8 Quadrilaterals

Solution

1. (a) 90°

Explanation:

Given : ABCD is a parallelogram in which AO and BO are angle bisectors of $\angle A$ and $\angle B$

Now since ABCD is a parallelogram

$\therefore AD \parallel BC$

Now $AD \parallel BC$ and transversal AB intersect them

$\therefore \angle A + \angle B = 180^\circ$ (\therefore sum of consecutive interior angle is 180°)

$$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle B = 90^\circ$$

$$\Rightarrow \angle 1 + \angle 2 = 90^\circ \text{ (}\therefore \text{AO and BO are angle bisectors) ... (1)}$$

In $\triangle AOB$ we have

$$\angle 1 + \angle AOB + \angle 2 = 180^\circ$$

$$\Rightarrow 90^\circ + \angle AOB = 180^\circ \text{ (from (1))}$$

$$\Rightarrow \angle AOB = 180^\circ - 90^\circ = 90^\circ$$

2. (a) $\angle A = 60^\circ, \angle C = 60^\circ, \angle B = 120^\circ$

Explanation:

$$\angle A = 60^\circ, \angle C = 60^\circ, \angle B = 120^\circ$$

Opposite angles are equal and sum of adjacent angles are supplementary.

3. (b) $AC^2 + BD^2 = 4AB^2$

Explanation: ABCD is a rhombus.

$$AB = BC = CD = DA$$

In Rhombus, diagonals bisect each other at right angles.

$$\text{So, } AO = CO \text{ and } BO = DO$$

In triangle AOB, $AO^2 + BO^2 = AB^2$ (Pythagoras theorem)

$$\left(\frac{1}{2} AC\right)^2 + \left(\frac{1}{2} BD\right)^2 = AB^2$$

$$AC^2/4 + BD^2/4 = AB^2$$

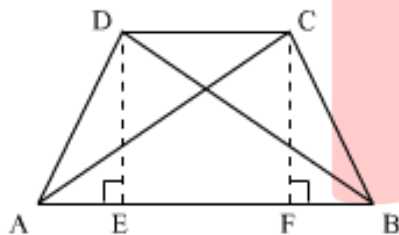
$$AC^2 + BD^2 = 4 AB^2$$

4. (d) $BC^2 + AD^2 + 2AB \cdot CD$

Explanation:

Given: ABCD is a trapezium with $AB \parallel CD$

Construction: Draw $DE \perp AB$ and $CF \perp AB$



Then in $\triangle ABC$

$\angle BAC$ is acute

$$\therefore BC^2 = AC^2 + AB^2 - 2 AF \cdot AB \dots(1)$$

and In $\triangle BDA$

$\angle DBA$ is acute

$$\therefore AD^2 = BD^2 + AB^2 - 2 BE \cdot AB \dots(2)$$

Adding (1) and (2) we get

$$BC^2 + AD^2 = AC^2 + BD^2 + 2AB^2 - 2AF \cdot AB - 2BE \cdot AB$$

$$\Rightarrow AC^2 + BD^2 = BC^2 + AD^2 - 2 AB [AB - AF - BE]$$

$$= BC^2 + AD^2 - 2AB [AB - (AE + EF) - (BF + EF)]$$

$$= BC^2 + AD^2 - 2AB [AB - (AE + EF + BF + EF)]$$

$$\begin{aligned}
 &= BC^2 + AD^2 - 2AB [AB - (AB + CD)] (\because EF = DC) \\
 &= BC^2 + AD^2 - 2AB [- CD] \\
 &= AD^2 + BC^2 + 2AB \times CD
 \end{aligned}$$

5. (d) $120^\circ, 60^\circ, 120^\circ, 60^\circ$

Explanation:

Let ABCD be a parallelogram and AP and CQ are the altitudes drawn from vertex A on sides DC and BC.

In quadrilateral APCQ, sum of the all angles = 360°

$$\text{So, } 60^\circ + 90^\circ + \angle C + 90^\circ = 360^\circ$$

$$\angle C = 360^\circ - 240^\circ = 120^\circ$$

$$\angle C + \angle B = 180^\circ \text{ (co-interior angles)}$$

$$\angle B = 180^\circ - 120^\circ = 60^\circ$$

In parallelogram, opposite angles are equal.

$$\text{So, } \angle A = \angle C = 120^\circ \text{ and } \angle B = \angle D = 60^\circ$$

6. congruent

7. $60^\circ, 120^\circ, 80^\circ, 100^\circ$

8. It need not be a parallelogram, because if $\angle A = \angle B = \angle C = 70^\circ$ and $\angle D = 150^\circ$. But $\angle B \neq \angle D$.

9. Let the fourth angle be x.

Sum of all angles of a quadrilateral = 360°

$$110^\circ + 50^\circ + 40^\circ + x = 360^\circ$$

$$\Rightarrow 200^\circ + x = 360^\circ$$

$$\Rightarrow x = 160^\circ$$

10. In $\triangle BEA$ and $\triangle CFA$,

$$BE = CF \dots\dots [\text{Given}]$$

$$\angle BEA = \angle CFA \dots\dots [\text{Each } 90^\circ]$$

$$\angle BAE = \angle CAF \dots\dots [\text{Common}]$$

$$\therefore \triangle BEA = \triangle CFA \dots\dots [\text{AAS axiom}]$$

$$\triangle AB = AC \dots\dots [\text{c.p.c.t.}]$$

Hence, $\triangle ABC$ is an isosceles.

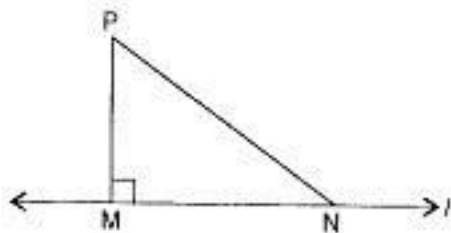
11. Given: l is a line and P is a point not lying on l .

$$PM \perp l$$

N is any point on l other than M

To Prove : $PM < PN$

Proof: In $\triangle PMN$, $\angle M = 90^\circ$



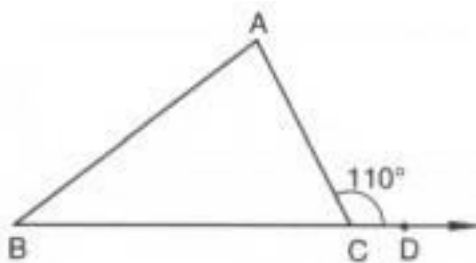
$\therefore \angle N$ is an acute angle.....[Angle sum property of a triangle]

$$\therefore \angle M > \angle N$$

$\therefore PN > PM \dots\dots [\text{Side opposite to greater angle is greater}]$

$$\therefore PM < PN$$

12.



Let ABC be a triangle whose side BC is produced to form an exterior angle $\angle ACD$ such that ext. $\angle ACD = 110^\circ$.

Let $\angle B = 30^\circ$. By exterior angle theorem, we have

$$\text{ext. } \angle ACD = \angle B + \angle A$$

$$\Rightarrow 110^\circ = 30^\circ + \angle A$$

$$\Rightarrow \angle A = 110^\circ - 30^\circ = 80^\circ$$

In $\triangle ABC$, we have

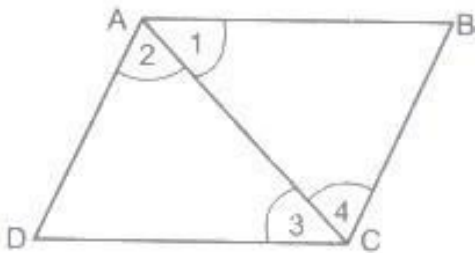
$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 80^\circ + 30^\circ + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 180^\circ - (80^\circ + 30^\circ) = 70^\circ$$

Hence, the other two angles of the triangle are 80° and 70°

13. ABCD is a parallelogram and diagonal AC bisect $\angle A$. We have to show that ABCD is a rhombus.



$$\angle 1 = \angle 2 \dots(1) [\because AC \text{ bisect } \angle A]$$

$$\angle 2 = \angle 4 \dots(2) [\text{Alt. interior angles}]$$

From (1) and (2), we get

$$\angle 1 = \angle 4$$

Now, in $\triangle ABC$, we have

$$\angle 1 = \angle 4 [\text{Proved above}]$$

$\therefore BC = AB$ [\because Side. Opp. To equal \angle s are equal]

Also, $AB = DC$ and $AD = BC$ [\because Opposite sides of a parallelogram are equal]

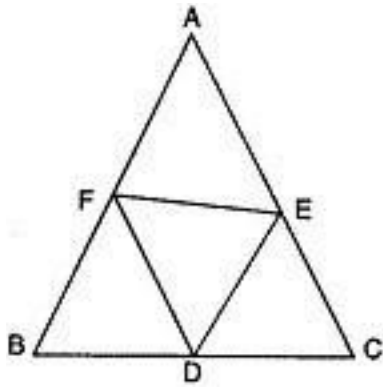
So, ABCD is a parallelogram in which its sides $AB = BC = CD = AD$.

Hence, ABCD is a rhombus.

14. Given : D, E and F are respectively the mid-points of sides BC, CA and AB of an equilateral triangle ABC.

To Prove : $\triangle DEF$ is also an equilateral triangle.

Proof :



Since line segment joining the mid-points of two sides of a triangle is half of the third side. Therefore, D and E are mid-points of BC and AC respectively.

$$\Rightarrow DE = \frac{1}{2} AB \text{ --- (i)}$$

E and F are the midpoints of AC and AB respectively.

$$\therefore EF = \frac{1}{2} BC \text{ --- (ii)}$$

F and D are the midpoints of AB and BC respectively.

$$\triangle FD = \frac{1}{2} AC \text{ --- (iii)}$$

Now, $\triangle ABC$ is an equilateral triangle.

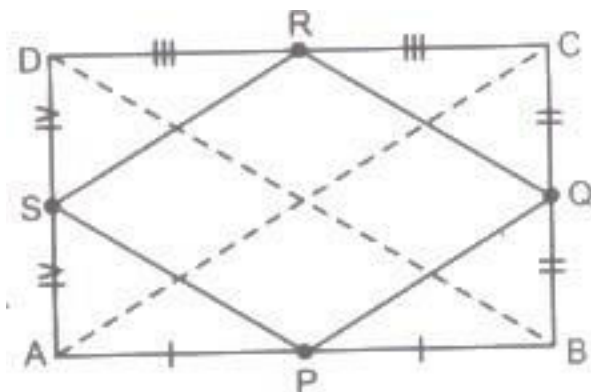
$$\Rightarrow AB = BC = CA$$

$$\Rightarrow \frac{1}{2} AB = \frac{1}{2} BC = \frac{1}{2} CA$$

$$\Rightarrow DE = EF = FD \text{ [using (i), (ii), (iii)]}$$

Hence, DEF is an equilateral triangle.

15. Given: A quadrilateral ABCD in which $AC = BD$ and P, Q, R and S are respectively the mid-point of the sides of AB, BC, CD and DA of quadrilateral ABCD.



To prove: PQRS is a rhombus

Proof: In $\triangle ABC$, P and Q are the mid-points of AB and BC respectively, i.e., PQ joins mid-points of sides AB and BC.

$$\therefore PQ \parallel AC \dots\dots\dots(1)$$

$$\text{And } PQ = \frac{1}{2}AC \text{ [by Mid-point theorem] } \dots\dots\dots (2)$$

In triangle ADC, R and S are the mid-points of CD and AD respectively.

$$\therefore SR \parallel AC \dots\dots\dots(3)$$

$$\text{And } SR = \frac{1}{2}AC \dots(4) \text{ [by Mid-point theorem]}$$

From (1) and (3), we obtain

$$PQ \parallel SR$$

From (2) and (4), we obtain

$$PQ = RS$$

\Rightarrow PQRS is a parallelogram.

In triangle ABD, join mid-points of sides of DA and AB respectively i.e; join SP.

$$\therefore SP = \frac{1}{2}BD \dots\dots\dots(5) \text{ [Mid-point theorem]}$$

$$\text{Since, } AC = BD \dots\dots\dots (6) \text{ [Given]}$$

From equations (2), (5) and (6), we obtain

$$SP = PQ$$

Since adjacent sides of parallelogram are equal, so, Parallelogram PQRS is a rhombus.

Hence, proved.

