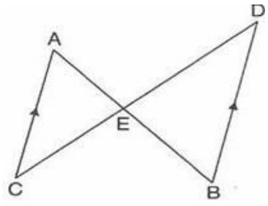
CBSE Test Paper 04

Chapter 6 Triangles

1. In the adjoining figure AC || BD. If , EB = 4 cm, ED = 8 cm, AC = 6 cm, AE = 3 cm then CE and BD are respectively **(1)**



   a. 5 cm, 7 cm.
   b. 7.5 cm, 9.5 cm.
   c. 6 cm, 8 cm.
   d. 4 cm, 6 cm.

2. A street light is fixed on a pole 6 m above the ground. If a woman of height 1.5 m casts a shadow of 3, then distance between her and the base of the pole is______. **(1)**
   a. 12 m
   b. 9 m
   c. 8 m
   d. 10 m

3. In an equilateral $\Delta ABC,\ AD \perp BC\ and\ AD^2 = p.\ BC^2$, then p is equal to **(1)**
   a. $\frac{1}{2}$
   b. $\frac{3}{4}$
   c. $\frac{2}{3}$
   d. $\frac{1}{3}$

4. A street light is fixed on a pole 6 m above the ground. If a woman of height 1.5 m casts a shadow of 3, then distance between her and the base of the pole is **(1)**
   a. 12 m.
   b. 8 m.
   c. 9 m.
   d. 10 m.

5. ABCD is a trapezium in which AB || DC and AB = 2DC. Diagonals AC and BD intersect at O. If $ar(\Delta AOB)$ = 84 cm^2, then $ar(\Delta COD)$ is equal to **(1)**

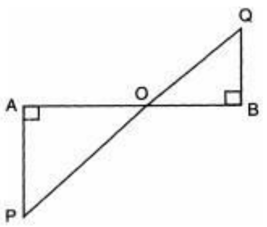
a. 24 cm^2

b. 42 cm^2

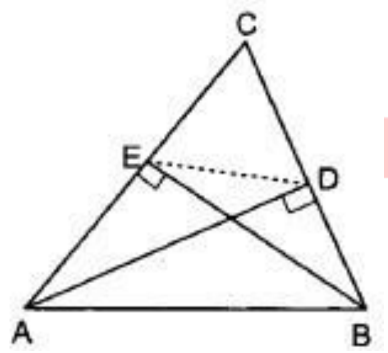
c. 28 cm^2

d. 21 cm^2

6. In the given figure, if $\angle A = 90^o, \angle B = 90^o$, OB = 4.5 cm, OA = 6 cm and AP = 4 cm, then find QB. **(1)**

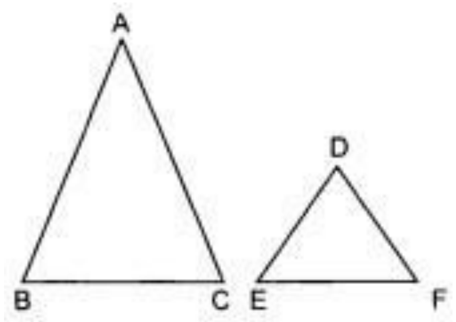


7. In Fig. AD and BE are respectively perpendiculars to BC and AC. Show that $\Delta ADC \sim \Delta BEC$ (1)

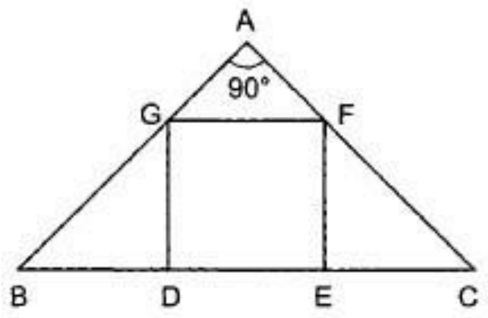


8. If $\triangle$ ABC ~ $\triangle$DEF such that 2AB = DE and BC = 6 cm, find EF. **(1)**

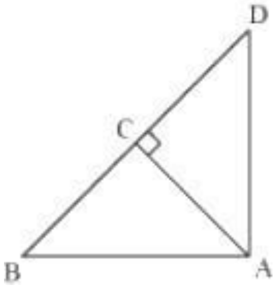
9. In the given figure, $\triangle$ ABC ~ $\triangle$ DEF. If AB = 2DE and area of $\triangle$ABC is 56 sq. cm, find the area of $\triangle$DEF. **(1)**



10. In Fig. DEFG is a square and $\angle$BAC = 90°. Prove that $\Delta AGF \sim \Delta DBG$ **(1)**

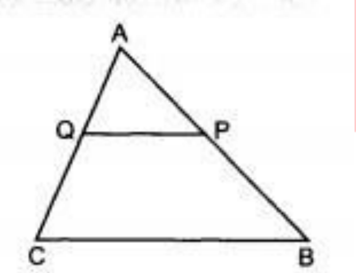


11. $\triangle$ABD is a right triangle right-angled at A and AC$\perp$ BD. Show that AB^2 =BC $\times$ BD **(2)**

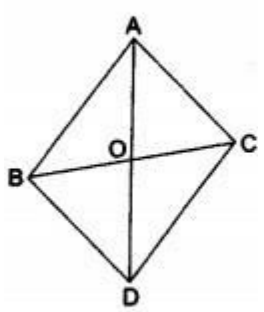


12. In ΔABC, P and Q are points on sides AB and AC respectively such that $PQ||BC$. If AP = 4 cm, PB = 6 cm and PQ = 3 cm, determine BC. **(2)**

13. In the fig., P and Q are points on the sides AB and AC respectively of $\triangle$ABC such that AP = 3.5 cm, PB = 7 cm, AQ = 3 cm and QC = 6 cm. Find BC. **(2)**



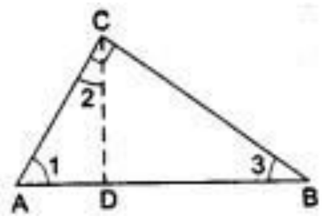
14. In the given figure, $\triangle ABC$ and $\triangle DBC$ are on the same base BC. If AD intersects BC at O, Prove that $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{AO}{DO}$. **(3)**



15. In a quadrilateral ABCD, P,Q,R,S are the mid-points of the sides AB, BC, CD and DA

respectively. Prove that PQRS is a parallelogram. **(3)**

16. In the given figure, $\angle ACB = 90^\circ$ and $CD \perp AB$. Prove that $CD^2 = BD \cdot AD$. **(3)**

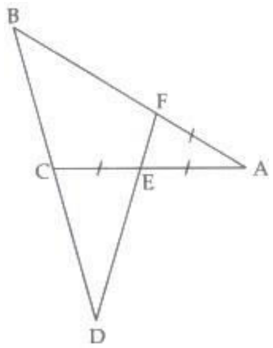


17. From an airport, two aeroplanes start at the same time. If speed of first aeroplane due North is 500 km/h and that of other due East is 650 km/h then find the distance between the two aeroplanes after two hours. **(3)**

18. ABCD is a trapezium in which AB || DC and P and Q are points on AD and BC, respectively such that PQ || DC. If PD = 18 cm, BQ = 35 cm and QC = 15 cm, find AD. **(4)**

19. Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides. **(4)**

20. In the given figure, line segment DF intersect the side AC of a triangle $\triangle$ ABC at the point E such that E is the mid–point of CA and $\angle$AEF = $\angle$AFE. Prove that: **(4)**
$\frac{BD}{CD} = \frac{BF}{CE}$.
[Hint: Take point G on AB such that CG $\|$ DF.]



CBSE Test Paper 04

Chapter 6 Triangles

Solution

1. c. 6 cm, 8 cm.

   Explanation: Given: AC || BD and AC = 6 cm, AE = 3 cm, EB = 4 cm, ED = 8 cm, In triangles ACE and DEB, $\angle$AEC = $\angle$DEB [Vertically opposite angles] $\angle$ ECA = $\angle$EDB [Alternate angles as AC || BD]

   $\therefore \triangle$ACE $\sim \triangle$DEB [AA similarity]

   $\therefore \frac{\mathrm{EB}}{\mathrm{AE}} = \frac{\mathrm{ED}}{\mathrm{EC}}$

   $\Rightarrow \frac{4}{3} = \frac{8}{\mathrm{EC}}$

   $\Rightarrow$ EC = $\frac{8\times3}{4}$ = 6 cm

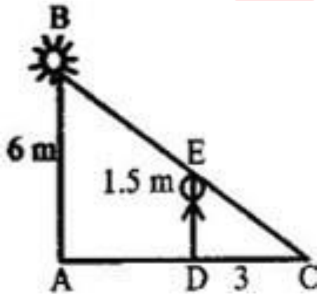
   Also $\frac{\mathrm{EB}}{\mathrm{AE}} = \frac{\mathrm{BD}}{\mathrm{AC}}$

   $\Rightarrow \frac{4}{3} = \frac{\mathrm{BD}}{6}$

   $\Rightarrow$ BD = $\frac{4\times6}{3}$ = 8 cm

2. b. 9 m

   Explanation: In triangles ABC and DEC,

   $\angle$A = $\angle$D [Each 90°]

   $\angle$C = $\angle$C [Common]

   Therefore, $\Delta\mathrm{ABC} \sim \Delta\mathrm{DEC}$ [AA similarity]

   $\Rightarrow \frac{\mathrm{AB}}{\mathrm{DE}} = \frac{\mathrm{AC}}{\mathrm{DC}}$

   $\Rightarrow \frac{6}{1.5} = \frac{\mathrm{AC}}{3} \Rightarrow$ AC = 12 m

   Therefore, the distance between the woman and pole is 12 - 3 = 9 m

3. b. $\frac{3}{4}$

   Explanation: In an equilateral triangle, ABC, if AD $\perp$ BC,

   Then $\mathrm{AB}^2 = \mathrm{AD}^2 + \mathrm{BD}^2$

$\Rightarrow \mathrm{AB}^2 = \mathrm{AD}^2 + \left(\frac{\mathrm{BC}}{2}\right)^2$
$\Rightarrow \mathrm{BC}^2 = \mathrm{AD}^2 + \frac{\mathrm{BC}^2}{4}$
$\Rightarrow \mathrm{AD}^2 = \mathrm{BC}^2 - \frac{\mathrm{BC}^2}{4}$
$\Rightarrow \mathrm{AD}^2 = \frac{3}{4}\mathrm{BC}^2$
Comparing with $\mathrm{AD}^2 = p.\mathrm{BC}^2$
$p = \frac{3}{4}$

4. c. 9 m.

Explanation: AB - Lamp post and DE - Woman

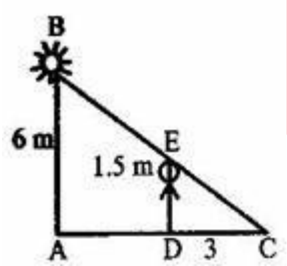
In triangles ABC and DEC,

$\angle$A = $\angle$D [Each 90^o]

$\angle$C = $\angle$C [Common]

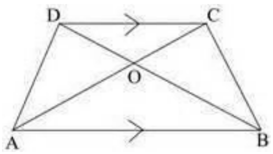
$\therefore \Delta\mathrm{ABC} \sim \angle\mathrm{DEC}$ [AA similarity]
$\therefore \frac{\mathrm{AB}}{\mathrm{DE}} = \frac{\mathrm{AC}}{\mathrm{DC}}$
$\Rightarrow \frac{6}{1.5} = \frac{\mathrm{AC}}{3}$
$\Rightarrow$ AC = 12 m

Therefore, distance between woman and pole = AC – DC = 12 – 3 = 9 m



5. d. 21 cm^2

Explanation: In triangles, AOB and COD,



$\angle$AOB = $\angle$COD [vertically opposite angles]

$\angle$ABO = $\angle$CDO [Alternate angles]

Therefore, $\Delta\mathrm{AOB} \sim \Delta\mathrm{COD}$ [AA similarity]

$\Rightarrow \frac{\mathrm{area}(\Delta\mathrm{AOB})}{\mathrm{area}(\Delta\mathrm{COD})} = \frac{\mathrm{AB}^2}{\mathrm{DC}^2} = \frac{(2\mathrm{DC})^2}{\mathrm{DC}^2} = \frac{4}{1}$

$\Rightarrow \mathrm{area}\,(\Delta\mathrm{COD}) = \frac{84\times1}{4}$ = 21 cm^2

6. In $\triangle$PAO and $\triangle$QBO

$\angle$A = $\angle$B = 90^o

$\angle$POA = $\angle$QOB (Vertically Opposite Angle)

$\triangle$PAO ~ $\triangle$QBO,(by AA criteria)

$\frac{OA}{OB} = \frac{PA}{QB}$

or, $\frac{6}{4.5} = \frac{4}{QB}$

or, $QB = \frac{4 \times 4.5}{6}$

Therefore, QB = 3 cm

7. In Δ's ADC and BEC, we have

$\angle$ADC = $\angle$BEC = 90° [Given]

$\angle$ACD= $\angle$BCE [Common]

So, by AA-criterion of similarity, we obtain

$\Delta ADC \sim \Delta BEC$

8. Given that $\triangle$ABC ~ $\triangle$DEF

We know that when two triangles are similar, then the ratios of the lengths of their corresponding sides are equal.

$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF}$

$\Rightarrow \frac{1}{2} = \frac{6}{EF}$

$\Rightarrow$ EF = 12 cm

9. $\triangle \mathrm{ABC} \sim \triangle \mathrm{DEF}$

Since for similar triangles,the ratio of the areas is the square of the ratio of their corresponding sides.

$\therefore \frac{\text{Area } \triangle \mathrm{ABC}}{\text{Area } \triangle \mathrm{DEF}} = \frac{\mathrm{AB}^2}{\mathrm{DE}^2}$

$\Rightarrow \frac{56}{\text{area } \Delta \mathrm{DEF}} = \frac{(2\mathrm{DE})^2}{\mathrm{DE}^2}$

$\Rightarrow \frac{56}{area \triangle DEF} = 4$

$\Rightarrow$ area $\triangle$DEF = $\frac{56}{4}$ = 14 cm^2

10. In ΔAGF and ΔDBG, we have

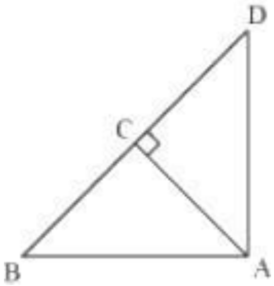
$\angle$GAF = $\angle$BDG [Each equal to 90°]

and, $\angle$AGF = $\angle$DBG [Corresponding angles]

$\therefore$ ΔAGF ~ ΔDBG [By AA-criterion of similarity]

11. Given: $\triangle$ABD is a right triangle right-angled at A and AC$\perp$ BD.

To Prove: AB2 =BC $\times$ BD



Proof: In $\triangle$ ADB and $\triangle$ CAB

$\angle DAB = \angle ACB = 90^\circ$

$\angle ABD = \angle CBA$ [common angle]

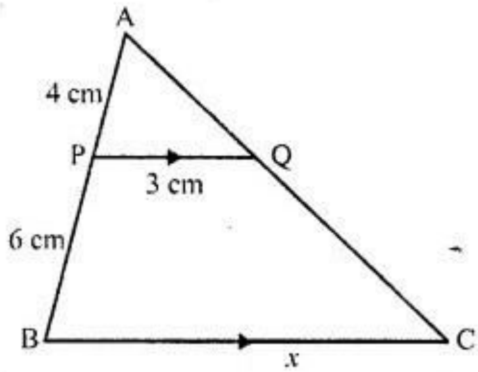
$\angle ADB = \angle CAB$ [remaining angle]

So, $\triangle$ ADB $\sim$ $\triangle$ CAB (by AAA similarity)

Therefore $\frac{AB}{CB} = \frac{BD}{AB}$

$\Rightarrow$ AB^2 = CB $\times$ BD

12. Let BC = x cm



In Δ's APQ and ABC,we have,

$\angle A = \angle A$

$\angle APQ = \angle ABC$

Therefore,by AA criteria of similar Δ's ,we have,

$\because \mathrm{PQ} \| \mathrm{BC}$

$\therefore \Delta \mathrm{APQ} \sim \Delta \mathrm{ABC}$

$\therefore \frac{\mathrm{AP}}{\mathrm{AB}} = \frac{\mathrm{AQ}}{\mathrm{AC}} = \frac{\mathrm{PQ}}{\mathrm{BC}}$

$\Rightarrow \frac{\mathrm{AP}}{\mathrm{AP+PB}} = \frac{\mathrm{PQ}}{\mathrm{BC}}$

$\Rightarrow \frac{4}{4+6} = \frac{3}{x} \Rightarrow \frac{4}{10} = \frac{3}{x}$

$\Rightarrow x = \frac{10 \times 3}{4} = \frac{15}{2}$

$\therefore \text{BC} = \frac{15}{2}\text{cm} = 7.5 \text{ cm}$

13. According to question it is given that P and Q are points on the sides AB and AC respectively of $\triangle$ABC such that AP = 3.5 cm, PB = 7 cm, AQ = 3 cm and QC = 6 cm.

$\frac{\text{AP}}{\text{PB}} = \frac{3.5}{7} = \frac{1}{2}$.....(I)

$\frac{\text{AQ}}{\text{QC}} = \frac{3}{6} = \frac{1}{2}$ .....(ii)

from (i) and (ii), we have

$\frac{\text{AP}}{\text{PB}} = \frac{\text{AQ}}{\text{QC}}$

$\therefore \quad \text{PQ} \| \text{BC}$

$\therefore$ $\angle$AQP = $\angle$ACB

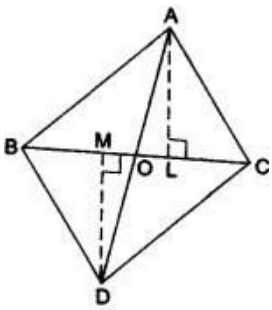
and $\angle$APQ = $\angle$ABC (corresponding angles)

$\therefore \Delta\text{AQP} \sim \Delta\text{ACB}$ (AA similarity)

$\Rightarrow \quad \frac{\text{PQ}}{\text{BC}} = \frac{\text{AQ}}{\text{AC}} = \frac{\text{AQ}}{\text{AQ+QC}}$ (By definition of SSS similarity)

$\Rightarrow \quad \frac{4.5}{\text{BC}} = \frac{3}{9}$ $\Rightarrow$ BC = 13.5 cm

14. Given: $\triangle ABC$ and $\triangle DBC$ are on the same base BC and AD intersects BC at O.



To Prove $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{AO}{DO}$.

Construction Draw $AL \perp BC$ and $DM \perp BC$.

Proof: In $\triangle ALO$ and $\triangle DMO$, we have

$\angle ALO = \angle DMO = 90^\circ$

and $\angle AOL = \angle DOM$ (vertical opposite angles)

$\therefore \triangle ALO \sim \triangle DMO$ [by AA-similarity]

$\Rightarrow \quad \frac{AL}{DM} = \frac{AO}{DO}$

$\therefore \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times BC \times DM} = \frac{AL}{DM} = \frac{AO}{DO}$ [using (i)]

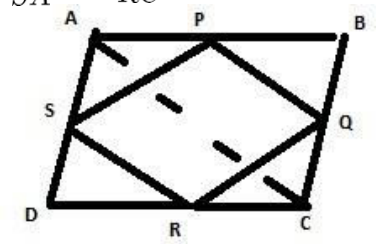
Hence, $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{AO}{DO}$

15. To Prove: PQRS is a parallelogram

Construction: Join AC

Proof: In △ DAC

$\frac{DS}{SA} = \frac{DR}{RC} = 1$



[∵ S and R are mid-points of AD and DC]

⇒ SR||AC .... (i) [by converse of B.P.T]

In △ BAC, $\frac{PB}{AP} = \frac{BQ}{QC} = 1$[∵ P and Q are midpoints of AB and BC]

⇒ PQ||PQ........(iii) [By converse of B.P.T]

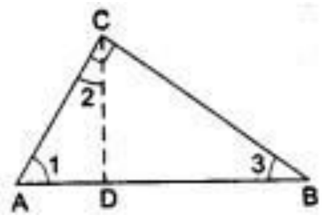
From (i) and (ii), we get

SR||PQ .... (iii)

Similarly, join B to D and PS||QR

⇒ ∴ PQRS is a parallelogram.

16. It is given that in $\triangle ABC$, $\angle ACB = 90^\circ$ and $CD \perp AB$.



To Prove: $CD^2 = BD \cdot AD$

Proof : In right $\triangle ADC$, we have

$\angle 1 + \angle 2 = 90^\circ$.....................(i)

In right $\triangle ACB$, we have

$\angle 1 + \angle 3 = 90^\circ$.........................(ii)

From (i) and (ii) we have

$\angle 1 + \angle 2 = \angle 1 + \angle 3$

$\Rightarrow \angle 2 = \angle 3$

In $\triangle ADC$ and $\triangle CDB$, we have

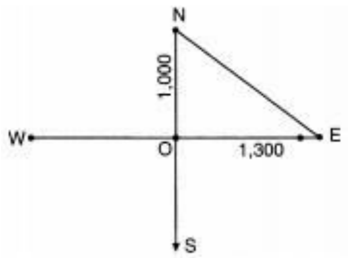
$\angle 2 = \angle 3$ (proved)

and $\angle ADC = \angle CDB = 90^\circ$

$\therefore \quad \triangle ADC \sim \Delta CDB$ [by AA-similarity]

$\therefore \quad \frac{AD}{CD} = \frac{CD}{BD}$

Hence, $CD^2 = BD \cdot AD$.

17. 

Distance covered by first aeroplane due North aftert two hours= ON = 500 $\times$ 2 = 1000 km

Distance covered by second aeroplane due East

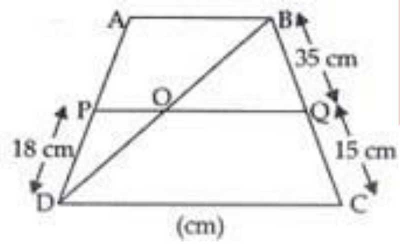
after two hours OE =$650 \times 2 = 1300$ km

As per shown in the figure the distance between the aeroplane will be equal to NE,Now in ΔONE

$NE = \sqrt{ON^2 + OE^2}$

$= \sqrt{1000^2 + 1300^2} = \sqrt{2690000} = 1640.12 km$

18. In trapezium ABCD



AB || CD (Given)

PQ || DC (Given)

and PD = 18 cm, BQ = 35 cm and QC = 15 cm

To find: AD

∴ AB || CD || PQ ...... (i)

In ΔBCD,

OQ || CD [From (i)]

$\therefore \frac{BO}{OD} = \frac{BQ}{QC}$ (ii) [By BPT]

Similarly, in ΔDAB,

PO || AB [From (i)]

$\therefore \frac{BO}{OD} = \frac{AP}{PD}$ (iii) [By BPT]

From (ii) and (iii)

$\frac{AP}{PD} = \frac{BQ}{QC}$

$\Rightarrow \frac{AP}{18} = \frac{35}{15}$

$\Rightarrow$ AP = $\frac{35}{15}$ × 18 = 7 × 6

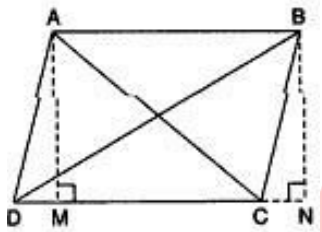
$\Rightarrow$ AP = 42 cm

∴ AD = AP + PD = 42 cm + 18 cm = 60 cm.

19. Given: ABCD is a parallelogram whose diagonals are AC and BD.

To prove: $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Construction: Draw AM ⊥ DC and BN ⊥ D(Produced)



Proof: In right triangle AMD and BNC.

AD = BC..............Opp.sides of a ||gm

AM = BN.............Both are altitudes of the same parallelogram to the same base

∴ △ AMD ≅ △ BNC ....RHS congruence criterion

∴ MD = NC ....(1) .... CPCT

In right triangle BND,

∵ ∠ N=90°

∴ $BD^2 = BN^2 + DN^2$................ By Pythagoras theorem

$= BN^2 + (DC + CN)^2$

$= BN^2 + DC^2 + CN^2 + 2DC.CN$

$= (BN^2 + CN^2) + CN^2 + 2DC.CN$

$= BC^2 + DC^2 + 2DC.CN$............(2)

In right triangle BNC with ∠N = 90°

$BN^2+CN^2 = BC^2$ ....By Pythagoras theorem

In right triangle AMC

∵ ∠ M=90°

∴ $AC^2 = AM^2 + MC^2$

$= AM^2 = (DC - DM)^2$

$= AM^2 + DC^2 + DM^2 - 2DC.DM$

$= (AM^2 + DC^2) + DC^2 - 2DC.DM$

$= AD^2 + DC^2 - 2DC.DM$

∵ In right triangle AMD with $\angle M = 90°$

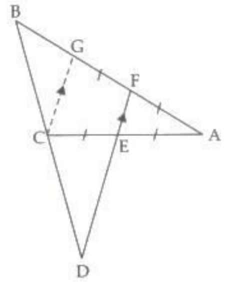
$AD^2 = AM^2 + DM^2$............[By Pythagoras theorem]

$= AD^2 + AB^2 - DC.CN$........ From(1)

Adding (3) and (2) ,we get

$AC^2 + BD^2 = (AD^2 + AB) + (BC^2 + DC) = AB^2 + BC^2 + BC^2 + CD^2 + DA^2$

20. To Prove: $\frac{BD}{CD} = \frac{BF}{CE}$



Construction: Draw CG || EF.

Proof: In $\triangle AGC\ \ CG \| EF$

∵ E is the mid point of AC

∴ F will be the mid point of AG.

$\Rightarrow FG = FA$

But, EC = EA = AF [Given]

$\therefore FG = FA = EA = EC$ ....(i)

In $\triangle BCG\ and \triangle BDF$

$EF \| CG$. (By construction)

$\therefore \frac{BC}{CD} = \frac{BG}{GF}$ [By BPT]

$\Rightarrow \frac{BC}{CD} + 1 = \frac{BG}{GF} + 1 \Rightarrow \frac{BC+CD}{CD} = \frac{BG+GF}{GF}$

$\Rightarrow \frac{BD}{CD} = \frac{BF}{GF}$

But, FG = CE [From (i)]

$\Rightarrow \frac{BD}{CD} = \frac{BF}{CE}$ Hence, proved.