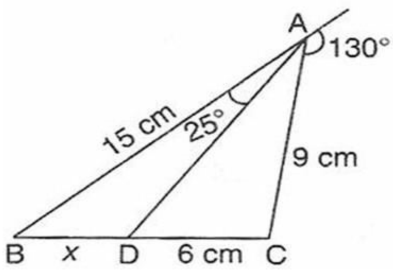
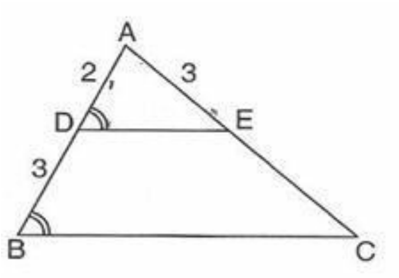
CBSE Test Paper 05

Chapter 6 Triangles

1. It is given that $\Delta ABC \sim \Delta DFE$, $\angle A = 30^0$, $\angle C = 40^0$, $AB = 5\ cm$, $AC = 8\ cm\ and\ DF =\ 7.5\ cm$. Then, the following is true. **(1)**
   a. $\angle F\ = 110^0, DE = 12cm$
   b. $\angle F\ = 40^0, DE = 12cm$
   c. $\angle D\ = 110^0, EF = 12cm$
   d. $\angle D\ = 30^0, EF = 12cm$
2. The areas of two similar triangles are respectively $121\ cm^2\ and\ 64\ cm^2$. If the median of the first triangle is 12.1 cm, then the corresponding median of the other triangle is equal to **(1)**
   a. 8.1 cm
   b. 8.8 cm
   c. 11 cm
   d. 11.1 cm
3. Two poles of height 8 m and 13 m are standing 12 m apart. The distance between their tops is______**(1)**
   a. 15 m
   b. 17 m
   c. 13 m
   d. 19 m
4. In the given figure, the value of x is:- **(1)**

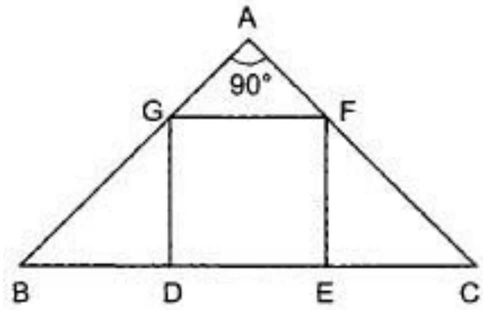


   a. 15 cm
   b. 10 cm
   c. 12 cm
   d. 6 cm
5. In the given figure if $\angle ADE\ = \angle ABC$, then CE is equal to **(1)**

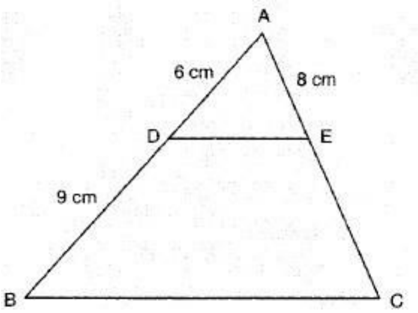


a. 2.
b. 4.5.
c. 5.
d. 3.

6. In Fig. DEFG is a square and ∠ BAC = 90°. Prove that $\Delta AGF \sim \Delta EFC$ **(1)**

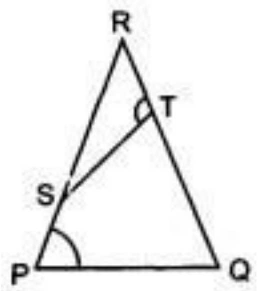


7. In Fig. DE || BC. Then, find AC. **(1)**

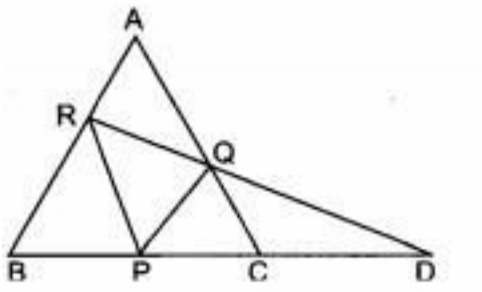


8. If $\triangle$ ABC and $\triangle$DEF are triangles such that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{4}{7}$. Find $\frac{\text{area} \triangle \text{ABC}}{\text{area} \triangle \text{DEF}}$. **(1)**

9. If $\angle \mathbf{P} = \angle RTS$, Then show that $\angle \text{PQR} = \angle \text{RST}$ **(1)**

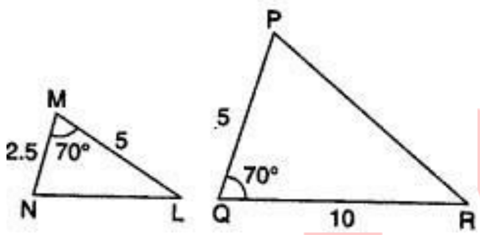


10. In the given figure PQ || BA; PR || CA. **(1)**



If PD = 12 cm, Find BD × CD.

11. State whether the pairs of triangles in the figure are similar or not. Write the similarity criterion used for answering the question and also write the pairs of similar triangles in the symbolic form. **(2)**



12. Two triangles DEF and GHK are such that ∠D = 48° and ∠H = 57°. If △DEF ~ △GHK then find the measure of ∠F. **(2)**

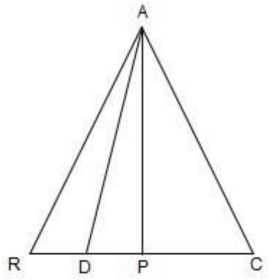
13. Let △ABC ~ △DEF. If ar (△ABC) = 100 cm^2 , ar (△DEF) = 196 cm^2 and DE = 7, then find AB. **(2)**

14. In a trapezium ABCD, diagonals AC and BD intersect at O. If AB = 3CD, then find ratio of areas of triangles COD and AOB. **(3)**

15. Prove that the area of the equilateral triangle described on the side of an isosceles right angled triangle is half the area of the equilateral triangle described on its hypotenuse. **(3)**

16. ABC is an isosceles triangle with AB = AC and D is a point on AC such that BC^2 = AC × CD. To prove: BD = BC. **(3)**

17. In an equilateral triangle ABC, D is a point on the side BC such the BD $= \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$. **(3)**



18. ABCD is a quadrilateral in which AD = BC. If P, Q, R, S be the mid-points of AB, AC, CD and BD respectively, show that PQRS is a rhombus. **(4)**

19. In Fig. $\angle BAC = 90^{\circ}$, AD is its bisector. If $DE \perp AC$, prove that DE $\times$ (AB + AC) = AB $\times$ AC. **(4)**

20. In an isosceles triangle ABC, if AB =AC= 13 cm and the altitude from A on BC is 5 cm, find BC. **(4)**

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Chapter 6 Triangles

Solutions

1. a. $\angle F = 110^0, DE = 12cm$

   Explanation: In ΔABC, $\angle$A + $\angle$B + $\angle$C = 180°

   $\Rightarrow 30^\circ + 40^\circ + \angle B = 180^\circ$

   $\Rightarrow \angle B = 110^\circ$

   Since $\Delta ABC \sim \Delta DFE$,

   therefore, $\angle B = \angle F = 110^\circ$

   Also $\frac{DF}{DE} = \frac{AB}{AC}$

   $\Rightarrow \frac{7.5}{DE} = \frac{5}{8}$

   $\Rightarrow$ DE = 12 cm

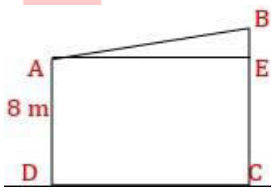
2. b. 8.8 cm

   Explanation: let x be the median of the other triangle , then by the theorem

   $\frac{121}{64} = \frac{(12.1)^2}{(x)^2} \Longrightarrow x = 8.8$

3. c. 13 m

   Explanation:



   In triangle ABE, AB = $\sqrt{AE^2 + BE^2} = \sqrt{(12)^2 + (13-8)^2} = \sqrt{144 + 25}$

   $\Rightarrow$ AB = 13 m

   Therefore, the distance between the tops of the pole is 13 m.

4. b. 10 cm

   Explanation: Here $\angle$CAD = $180^\circ - (130^\circ + 25^\circ) = 25^\circ$

   Since $\angle$CAD = $\angle$DAB, therefore, AD is the bisector of $\angle$BAC.

   Since internal bisector of an angle of a triangle divides the opposite sides internally in the ratio of the sides containing the angle.

   $\therefore \frac{BD}{DC} = \frac{AB}{AC}$

   $\Rightarrow \frac{x}{6} = \frac{15}{9}$

   $\Rightarrow x = 10$ cm

5. b. 4.5.

Explanation: In ΔABC and ΔADE,

$\angle$ADE = $\angle$ABC [Given]

$\angle$A = $\angle$A [Common]

$\therefore \Delta ABC \sim \Delta ADE$ [AA Similarity]

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{2}{3} = \frac{3}{EC}$

$\Rightarrow$ EC = 4.5 cm

6. In Δ AGF and Δ EFC, we have

$\angle$FAG = $\angle$CEF [Each equal to 90°]

and, $\angle$AFG = $\angle$ECF [Corresponding angles]

$\therefore \Delta AGF \sim \Delta EFC$ [using AA-criterion of similarity]

7. From figure,

DE || BC

Therefore, by Thales theorm ,we have,

$\frac{AD}{AB} = \frac{AE}{AC}$

AC = $\frac{AE \times AB}{AD}$

= $\frac{AE \times (AD+DB)}{AD}$

= $\frac{8 \times (6+9)}{6}$

= $\frac{8 \times 15}{6}$

= 20

8. $\because \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

$\therefore \quad \triangle ABC \sim \triangle DEF$

Since for similar triangles,the ratio of the areas is the square of their corresponding sides.

$$\Rightarrow \quad \frac{area \triangle ABC}{area \triangle DEF} = \frac{AB^2}{DE^2} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{4}{7}\right)^2 = \frac{16}{49}$$

9. In $\triangle$RPQ and $\triangle$ RTS

$\angle P = \angle RTS$ (Given)

$\angle R = \angle R$(common)

Therefore,by AA criteria of similar triangles, ΔRPQ ∼ ΔRTS.Therefore,

$\angle PQR = \angle RST$

10. In $\triangle$BRD,

BR $\|$ PQ

Therefore,by basic proportionality theorm,

$\frac{BD}{PD} = \frac{RD}{QD}$ ..(i)

In $\triangle$RDP, PR $\|$ QC (given)

Therefore,by basic proportionality theorm,

$\frac{RD}{QD} = \frac{PD}{CD}$ ..(ii)

from (i) and (ii)

$\frac{PD}{CD} = \frac{BD}{PD}$

$\Rightarrow \quad BD \times CD = PD \times PD = 12 \times 12$ = 144 cm^2

11. In $\triangle$MNL and $\triangle$QPR,

$\frac{ML}{QR} = \frac{MN}{QP} \left(= \frac{1}{2}\right)$ and $\angle NML = \angle PQR$

$\therefore \triangle MNL \sim \triangle QPR$........SAS similarity criterion

12. Given that $\triangle$DEF ∼ $\triangle$GHK.

$\angle$D =$\angle$G=48^o······(Given)

$\angle$E =$\angle$H=57^o·······(Given)

In $\triangle$DEF,

$\angle$D+$\angle$E+$\angle$F=180^o····(Angle sum property)

$\Rightarrow$ 48^o +57^o+$\angle$F=180^o

$\Rightarrow$$\angle$F=$75^o$

13. $\Delta ABC \sim \Delta DEF$,then

$\frac{ar(\triangle ABC)}{ar(\triangle DEF)} = \frac{AB^2}{DE^2}$ [as ,The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides]

or, $\frac{100}{196} = \frac{AB^2}{(7)^2}$

or, $\frac{100}{196} = \frac{AB^2}{49}$

or, $AB^2 = \frac{49 \times 100}{196}$

or, $AB^2 = 25$

$\therefore$ AB = 5 cm

14. In Δ A O B and Δ C O D

$\angle AOB = \angle COD$

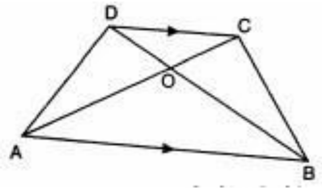
$\angle ABO = \angle CDO$

Hence $\Delta AOB \sim \Delta COD$

$\frac{\text{ar}(\Delta COD)}{\text{ar}(\Delta AOB)} = \frac{CD^2}{AB^2}$

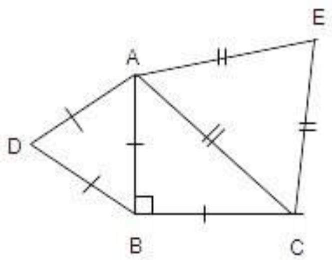
$= \frac{CD^2}{(3CD)^2} = \frac{CD^2}{9CD^2} = \frac{1}{9}$

ratio = 1 : 9



15. Given: In $\triangle ABC$ in which $\angle ABC = 90^o$ and AB = BC.

Also, $\triangle ABD$ and $\triangle ACE$ are equilateral triangles.



To prove : $ar(\triangle ABD) = \frac{1}{2} ar(\triangle CAE)$

Proof : Let AB = BC = x units

In $\triangle ABC$

By Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Now,$CA = \sqrt{AB^2 + BC^2}$

$= \sqrt{x^2 + x^2}$

$= \sqrt{2x^2} = x\sqrt{2}$

In $\triangle ABD$ and $\triangle CAE$, each angle is 60^o as they are equilateral triangles.

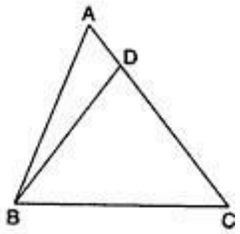
$\therefore \triangle ABD \sim \triangle CAE$

Since,the ratio of the area of two similar triangles is equal to the square of ratio of their corresponding sides.

$\therefore \frac{ar(\triangle ABD)}{ar(\triangle CAE)} = \frac{AB^2}{CE^2}$
$\therefore \frac{ar(\triangle ABD)}{ar(\triangle CAE)} = \frac{x^2}{(x\sqrt{2})^2}$
$\therefore \frac{ar(\triangle ABD)}{ar(\triangle CAE)} = \frac{x^2}{2x^2}$
$\therefore \frac{ar(\triangle ABD)}{ar(\triangle CAE)} = \frac{1}{2}$
Hence, $ar(\triangle ABD) = \frac{1}{2}ar(\triangle CAE)$

16. To prove: BD = BC

proof:BC^2 = AC $\times$ CD



$\Rightarrow \frac{AC}{BC} = \frac{BC}{CD}$ ....(1)

Also, $\angle$ ACB = $\angle$ BCD ....(2) ..... [Common angle]

In view of (1) and (2),

$\triangle$ ABC $\sim$ $\triangle$BDC ..... [SAS similarity criterion]

$\therefore \frac{AC}{BC} = \frac{AB}{BD}$ .... ($\because$ corresponding sides of two similar triangles are proportional)

But AB = AC .... (Given)

BD = BC

17. Given $\triangle ABC$ in which AB = BC = CA and BD = $\frac{1}{3}BC$.

Construction : Draw $AP \perp BC$

In $\triangle ADP$

$AD^2 = AP^2 + DP^2$

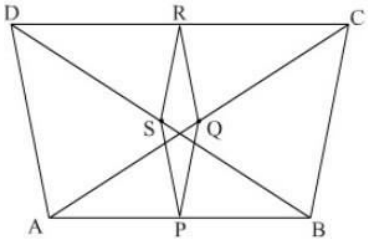
$AD^2 = AP^2 + (BP - BD)^2$

$AD^2 = AP^2 + BP^2 + BD^2 - 2BP \cdot BD$

$\Rightarrow AD^2 = AB^2 + \left(\frac{1}{3}BC\right)^2 - 2\left(\frac{BC}{2}\right)\left(\frac{BC}{3}\right)$

$AD^2 = AB^2 + \frac{AB^2}{9} - \frac{AB^2}{3}$
$[\therefore AP^2 + BP^2 = AB^2 \text{ from } \triangle APB]$
$AD^2 = \frac{7}{9}AB^2$
Hence, $9AD^2 = 7AB^2$

18. We have,



In ΔBAD, by mid-point theorem

PS||AD and $PS = \frac{1}{2}AD$ ....(i)

In ΔCAD, by mid-point theorem

QR||AD and $QR = \frac{1}{2}AD$ ....(ii)

Compare (i) and (ii)

PS || QE and PS = QR

Since one pair of opposite sides is equal as well as parallel then, PQRS is a parallelogram.....(iii)

Now, In ΔABC, by mid-point theorem,

PQ||BC and $PQ = \frac{1}{2}BC$ ....(iv)

and, AD = BC... (v) [Given]

Compare equations (i)(iv) and (v)

PS = PQ.... (vi)

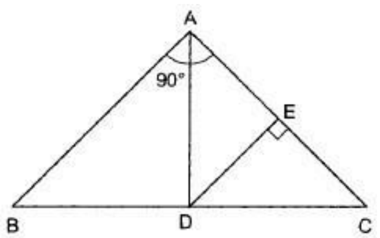
From (iii) and (vi),

Since, PQRS is a parallelogram with PS = PQ then PQRS is a rhombus.

19. To prove the given result,we will use the following theorm.

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle

Since AD is the bisector of $\angle$A of ΔABC.



$\therefore \quad \frac{AB}{AC} = \frac{BD}{DC}$ [by above theorm]

$\Rightarrow \quad \frac{AB}{AC} + 1 = \frac{BD}{DC} + 1$ [Adding 1 on both sides]

$\Rightarrow \quad \frac{AB+AC}{AC} = \frac{BD+DC}{DC}$

$\Rightarrow \quad \frac{AB+AC}{AC} = \frac{BC}{DC}$ ... (i)

In Δ's CDE and CBA, we have

$\angle$DCE = $\angle$BCA = $\angle$C [Common]

$\angle$BAC = $\angle$DEC [Each equal to 90°]

So, by AA-criterion of similarity, we have

ΔCDE ~ ΔCBA

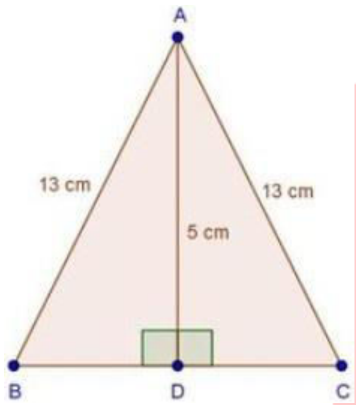
$\Rightarrow \quad \frac{CD}{CB} = \frac{DE}{BA}$

$\Rightarrow \quad \frac{AB}{DE} = \frac{BC}{DC}$ ...(ii)

From (i) and (ii), we obtain

$\frac{AB+AC}{AC} = \frac{AB}{DE} \Rightarrow DE \times (AB + AC) = AB \times AC$

20. In ΔADB, by pythagoras theorem,



$AD^2 + BD^2 = AB^2$

$\Rightarrow 5^2 + BD^2 = 13^2$

$\Rightarrow 25 + BD^2 = 169$

$\Rightarrow BD^2 = 169 - 25 = 144$

$\Rightarrow$ BD = $\sqrt{144}$ = 12 cm

In ΔADB and ΔADC

$\angle ADB = \angle ADC$ [Each 90°]

AB = AC [Each 13 cm]

AD = AD [Common]

Then, $\Delta ADB \cong \Delta ADC$ [By RHS condition]

$\therefore$ BD = CD = 12 cm [By c.p.c.t]

Hence, BC = 12 + 12 = 24 cm.